



NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Qualifications Framework Levels 1–3, 2005

Mathematics

National Moderator's Report

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General Guidance for Assessors of Achievement and Unit Standards

The purpose of external moderation is to provide reassurance that assessor judgments are at the national standard and are made on the basis of assessment materials that are fair and valid.

All assessment materials are expected to:

- give the learner the opportunity to meet the requirements of the standard
- have an assessment schedule that gives evidence of appropriate learner responses and clear judgments at all levels.

The Ministry of Education contracted subject experts to write assessment resources for achievement standards. These are not pre-moderated. The intention is that they are modified to suit teaching programmes and learner needs. They do not provide 'rules' but suggest different ways of assessing to the nationally registered standard.

General Overall Comment

The major issue with material sent for moderation was schools submitting only tick sheets rather than schedules completed with appropriate student responses. In the case of measurement tasks, or where the candidate was required to select data from a list that had been supplied, an acceptable range of values was expected and frequently not supplied.

Most internally assessed achievement standards have a practical component. Many providers have modified web tasks to remove the practical aspect which has made them invalid. When creating their own tasks, schools must pay attention to the details of the standards.

Where standards involve solving problems, learners are expected to choose the technique to use.

Some providers are using older versions of the standards, which frequently disadvantage their students.

Where students are required to generate random numbers it is not necessary for them to describe the calculator process involved.

Excellence is still the major problem across the standards, with scanty and rote type comments being accepted. These are frequently not linked to the context.

In some cases where AS 90193 and AS 90288 were assessed together in the one assessment, the level 2 standard was misinterpreted. By answering their question learners had difficulty making an inference.

The use of appropriate technology is expected in all internal achievement standards. This is either stated or assumed for all standards. Some schools are still using ordinary calculators as the only technology, even for level 3 statistics. This is unfair for such learners.

AS 90149: *Solve problems involving the measurement of everyday objects*

Generally the tasks for this standard were at a more appropriate standard this year. Where issues arose they related to the following points.

Achievement level

- The standard requires learners to determine the measurements they need and take these in order to solve a problem that uses real objects.
- Learners must correctly solve at least one problem using their own measurements.
- Scale drawings and drawings of objects are not acceptable.

Merit level

- The conversions, etc, that are listed in the explanatory notes must be used within the context of solving a problem.

Excellence level

- The models need to be more complex than involving packaging of rectangles into rectangular boxes.

The main issues related to the assessment schedule. As the school chooses the objects that learners are to measure and the equipment they will use, they must include the actual measurements on their schedules and the range of answers that are acceptable.

AS 90150: Use geometric techniques to produce a pattern or object

The assessment tasks for this standard were generally appropriate. However, teachers frequently failed to recognise evidence for Merit that was presented in writing instructions for Excellence. Having provided the evidence in the Excellence questions, candidates are still eligible for the award of Excellence.

For Merit, the representations are to be combined in the production of an object or a model. For Excellence, it is now clear that writing the instructions to produce *their* object or pattern is intended. Evaluation is no longer included.

Most successful learners are those who keep their designs simple although they must include at least two constructions and at least two transformations.

For transformations, learners must specify the:

- distance and direction of the movement (for translations)
- mirror line (for reflection)
- centre, angle of rotation, and the direction (for rotation)
- centre and scale factor (for enlargement).

For constructions, learners must specify:

- the type of construction
- the position
- any direction, if appropriate.

AS 90193: Use statistical methods and information

This standard was generally well-assessed but by using Version 1 of the standard candidates were disadvantaged as Excellence now only has one criterion. Excellence involves an evaluation of the learner's process.

Comments must relate to:

- what the learner has done, rather than speculating on aspects about data selection/collection, etc
- factors such as limitations of the learner’s process – this could include comments on data collection if learners have collected their own data or if details on the data collection have been given
- discussion of what they have done and why
- the effectiveness, or accuracy, of their selection of data analysis and display reasons why the particular statistics have been calculated and why particular graphs were drawn, and justify the selection
- suggested improvements, comments on further analysis or displays that would enhance the strength of the candidate’s response
- possible aspects that could be explored to support or extend the investigation - this could include further data that would have enhanced the learner’s response.

Questions posed for investigation should be specific, eg “Does the tariff for a standard hotel room for two people in Auckland for one night differ from the tariff for a standard hotel room for two people in Wellington for one night?”. While the question may start more generally, eg “Does the cost of hotel accommodation in Auckland differ from the cost of hotel accommodation in Wellington?”, it needs to be made more specific at some point. This is to ensure comparisons that are made are valid, ie that “like things are being compared”, and to avoid the situation in which different conclusions could be justified. In the hotel example, the data should exclude ‘specials’ and discounts for extended stays, etc.

The basis of the comparison should also be made clear and justified. In the hotel example, the **mean** daily tariff for a standard room in an Auckland hotel may be less than the mean daily tariff for a standard room in a Wellington hotel; but the reverse may be true for the **median** tariffs. The mean provides the principal basis for comparison because it uses the most information provided by the data, ie the actual values, whereas the median uses only the position of the data, although the median may be used if the data had extreme values that distorted the data.

AS 90288: *Select a sample and use this to make an inference about a population*

Some assessors rejected samples of less than 30 where a sample of, say 25, was just as valid. This is one of the situations where it is necessary to look back at the intention of the standard.

Achievement

- Scaffolding trivialises the task; it is inappropriate to direct the student to calculate statistic(s), eg candidates need to understand that the calculation of a statistic is required in order to make an inference.
- Learners are required to make an inference about the population from their analysis of their sample.
- Learners must use their sample in their calculations to make an inference.
- Too often, candidates did not relate their inference to the context or even acknowledge that it was an estimate. Learners need to understand that an inference made from sampling must relate to the population that was sampled, eg “all students at that school” and not “all students”, and that the inference must be clearly stated, eg not just “average weight of school bags is . . .” but “mean weight of school bags for all students at X School is”.

- Acceptable measures of spread are standard deviation, variance and inter-quartile range. Stating the range, or the value of the maximum and minimum, or the upper and lower quartile is not sufficient.
- One statistic is sufficient but must be appropriate to the inference being made, eg median could be used to estimate population mean.
- For stratified random sampling, the estimate of the population should be found from the statistic calculated for the entire sample. If the candidate has calculated the statistics for each of the strata and these are not of the same size, then combining these to find the population estimate needs to be weighted.

Merit

- Learners most commonly make a comment such as “must be representative because I used systematic random sampling”. This was accepted. No attempt was required to relate the sample back to characteristics of the population.
- Some markers accepted woolly speculation rather than some further analysis of the data.
- Justification in Merit – must relate back to the underlying population or the purpose of the investigation, eg stratified so males and females could be compared.
- For Merit, learners must describe their sampling method. However, they do not need to describe how to use their tool in selecting their sample, ie learners are not required to explain how to generate random numbers using their calculator or computer.

Excellence

- Very few scripts had comments in enough depth for Excellence, though students were given this grade. Speculation and vague ideas, without anything to back them up from the data, were accepted.
- For Merit/Excellence, the distinction comes from the requirement to critically evaluate the sampling process and the need for learners to consider the implication of any limitations on the inference they have made.

Schedules must include examples of statements that are valid for evaluations. The statements must relate to the investigation and the learner’s analysis, or results, rather than rote responses.

AS 90289: Simulate probability situations, and apply the normal distribution

Often the descriptions of simulations for Achievement were poor as they did not contain sufficient details for the simulation to be replicated. There was also a failure to recognise that, for Merit, both theoretical and simulations problems need to be solved. This led to inappropriate grades being awarded.

In dealing with simulations in this standard, teachers need to ensure that learners understand that a simulation of a phenomenon is usually conducted:

- when there is no known mathematics to describe the phenomenon, or
- some mathematics for the phenomenon has been developed but it is very complex or incomplete, eg there may be no known way of solving equations in the mathematical theory
- a comparison with a theoretical prediction is usually not possible (a simulation may be carried out to test a theory, in which case a comparison with the theory may be possible).

Simulations are usually trialled over many runs (possibly hundreds or thousands) and are therefore usually run using computer-generated data.

AS 90291: Solve trigonometry problems requiring modeling of practical situations

For this standard, candidates are required to take measurements and solve a trigonometry problem. Scale drawings and maps are not appropriate. For Achievement, candidates may use right angle triangle techniques to solve the non-right angle triangle problem. The range of acceptable answers/measurements must be provided in the schedule. These vary from situation to situation. General Explanatory Note 3 requires learners to relate their answer to the context and apply appropriate rounding.

AS 90637: Solve problems and equations involving trigonometric functions

There were few problems here.

- The interpretation of the explanatory note is, “For Achievement either C or D is 0 and A and B are not both = 1”.
- While sine is used in the first example given in the explanatory notes for Achievement, other trig functions are acceptable and expected.
- Errors that arise from having the calculator in the incorrect mode should be penalised.
- For Merit, the learner must determine A and B and 1 of C or D.
- Learners should be expected to use both radians and degrees.

Level 3 Statistics

The model chosen was often inappropriate for the data. Learners have not been provided with sufficient information about the data collection to be able to make appropriate comments. Correlation coefficients and R^2 values are often being confused.

In discussing the appropriateness of a model, students should not rely primarily on the R^2 value. A decision about the appropriateness of a model should be made primarily from an understanding about the nature of the variables, eg an exponential model is appropriate for variables for which $\frac{dy}{dx} \propto x$, and/or the scatter graph with the regression line fitted, and may be **supported by** the R^2 value. It is common for learners to use all the forms of model available to them in Excel and to make a decision about the appropriateness of a model based entirely on the R^2 value, sometimes with insignificant differences between them.

An R^2 value, ie a value of the coefficient of determination, is appropriate for any form of regression model. However, a correlation coefficient is only appropriate for a linear model. Some candidates cite a correlation coefficient for a non-linear model.

The interpretation of R^2 requires care. It measures the proportion of the total variability (about the mean) of the response variable that is explained, or accounted for, by the regression effect between the explanatory variable and the response variable. Some learners say that it measures the proportion of the variability in the response variable that is explained by, or due to the variability in, the explanatory variable, but the definition of R^2 contains no reference to the explanatory variable. Note the response variable is sometimes called the ‘predicted variable’ and the explanatory variable the ‘predictor variable’.

AS 90641: Determine the trend for time series data

The following evidence is expected for this standard at Achievement, Merit and Excellence levels.

Achievement

- Trend line should be shown on graph of raw and smoothed data.

- Gradient (or equation) quantitatively interpreted in context.

There were frequently poor descriptions of trends given which were more appropriate for level 1. “The trend is increasing” is not sufficient for a description.

Merit

- Time series worksheet including individual (quarterly) effects – estimates of seasonal effect for the forecast season.
- Equation of trend line and this is used to obtain the trend component for the forecast (or trend component read from an appropriate graph).
- Forecast, made from trend line and seasonal effect, and quoted in context.
- Other methods of obtaining the trend **may** be appropriate but the validity of these must be checked, eg extending of table.

Excellence

- For Excellence, comments were accepted which were lightweight. Comments also need to be distinct. Accepting the same idea three times in different wording is not sufficient.
- A coherent report is expected relating to the requirements set for each of the bullet points. This must not contain significant contradictions or statistical misunderstandings. The report must flow and be logical. It may be in bullet point form though paragraph form is preferred.
- Any statement for Excellence must be exemplified by and linked to the context.
- Comments of a trivial or wildly speculative nature are not acceptable.
- Consideration as to whether the model is additive, multiplicative, step, ramp and why – this needs to be a comment related to the smoothed data. Basing a decision on the R^2 value alone is not appropriate. It is primarily visual.
- Seasonally adjusted data shown. Discussion involving interpretation of seasonally adjusted values – related to seasonal adjustment being a form of smoothing. This may involve comparing seasonally adjusted data with the trend line.
- Relate the results to the context, eg reference is made to whom it is useful for – economic planners or individual business. It could include reference to changes in conditions.
- A polynomial is rarely appropriate as a trend line if the implications are taken over unlimited time.
- Candidates need to consider the number of periods the analysis has been based on.
- Discussion of curvature, linearity, outliers, ramps, etc where appropriate.
- Background information, or the collection of own data, is required for a valid comment on this aspect. Positive statements relating to data collection are appropriate when the background information is provided.
- Consideration of the appropriateness of what has been done with justification is sufficient evidence for improvement. It may be appropriate to consider the use of different regression lines, eg equal weighting of data in moving averages methods.

Merit – Seasonally Adjusted Data

Seasonal adjustments are made by subtracting the estimate of the appropriate seasonal effect, ie the mean of the individual seasonal effects, from each observation. Seasonally adjusted data allow comparisons between consecutive time periods by removing the seasonal effects and leaving the trend component (and the error component). Seasonal adjustments provide a form of smoothing. It

was not uncommon for candidates to explain seasonal adjustments by comparing each seasonally adjusted value with the corresponding centred moving mean, and say whether the seasonally adjusted value was above or below expectations. This is not useful as centred moving means, per se, have no physical meaning in the context of time series. They are a ‘means to an end’, to enable the trend component of the series to be determined.

Excellence

For the following aspects for Excellence to be met satisfactorily – discussion of relevance and usefulness of forecast, and discussion of potential sources of bias – candidates need to be given information about the source of data and/or the data collection method; or if they collect the data themselves they must ensure they obtain such information. Otherwise, any comments become, at best, simply speculative.

Note that it is likely that potential sources of bias will have been minimised in any reputable data collection process.

AS 90645: Complete a statistical investigation involving bi-variate data

The following points should be noted for this standard.

- Where candidates have not collected their own data and are not provided with details of the data collection method, comments should relate to the analysis of the data and not the source of the data
- Candidates often did not explicitly describe the relationship. Often predictions were accepted as sufficient.
- The requirement for data to be continuous was not adhered to although it is specifically stated in the standard.
- Many assessors did not understand what the coefficient of determination, R^2 , was or how to use it correctly. The correlation coefficient, r , was also applied incorrectly in most cases.
- Many assessors accepted a simple substitution with no comments for the interpretation of the regression analysis and correlation coefficient.
- Many markers accepted the routine removal of outliers to ‘improve R^2 ’. Improvements were often judged simply on R^2 value with no discussion of whether the equation made any sense.
- Comments were accepted which were lightweight. Comments need to be distinct. Accepting the same idea three times in different wording is not sufficient.
- GEN 3 was ignored by some assessments.
- Generally, for the purpose of this standard, linear regression is sufficient. Non-linear regression models should be treated cautiously and must be justified.

At Achievement level, analysis:

- should include a scatter graph
- must include either a regression model **or** a correlation coefficient (correlation coefficient applies to linear relationships only).

The report will involve a statement of the relationship between one pair of variables. This must be quantified (correlation or regression) as well as involving a description of the relationship in context.

At Merit level, the following should be noted.

- The understanding of the dependent and independent variable is important. The appropriateness of a model should be primarily determined by a visual inspection of the scatter graph. It may be supported by a consideration of the R^2 value. The nature of the variables may also be a relevant consideration.
- Interpreting the regression coefficients (quantitatively), R^2 or the correlation coefficient in context. The coefficient of determination (R^2) measures the proportion of the variability in the response variable that is accounted for, or explained by, the regression.
- Predictions must be interpreted in context.
- Discussion must be in context and not involve contradiction.

At Excellence level, the following should be noted.

- A coherent report is expected relating to the requirements set for each of the bullet points. This must not contain significant contradictions or statistical misunderstandings. The report must flow and be logical. It may be in bullet point form though paragraph form is preferred.
- Any general statement for Excellence must be exemplified by, and linked to, the context.
- Comments at a trivial level are not acceptable.
- Consideration of the appropriateness of what has been done, with justification, is sufficient evidence for improvement. It may be appropriate to consider the use of different regression lines.
- Background information, or the collection of own data, is required for a valid comment on this aspect. Positive statements relating to data collection are appropriate when the background information is provided, eg how far is the extrapolation outside the range of the given data?
- Use of correlation or regression coefficients to make predictions.
- Statement of who may use this and why, eg the population to which this can be applied.

Merit

Outliers continue to be a problem. Candidates need to understand that a data value should be quite remote before being considered an outlier (although there is no consensus over when a data value should be treated as an outlier). Candidates commonly treat data on the margins of a data set as outliers, but they should be very cautious in classifying any data values as outliers.

In interpreting correlation coefficients and discussing the difference between correlation and causality, students should make clear reference to the context.

Excellence

To enable learners to discuss aspects required for Excellence in a meaningful way, teachers should make sure they are given information about the nature and source of data and/or the data collection method; or if they collect the data themselves students should ensure they obtain such information. Otherwise, comments tend to be purely speculative and are often meaningless.

In discussing how widely findings can be applied, candidates will need to know how any sampling was carried out. The findings can only be applied to the population from which the sampling was conducted, and may require other constraints. For example, if a survey was conducted on customers in a particular supermarket, any findings can only be applied to that supermarket at the time the survey was conducted, as the characteristics of the customers may be unique to that supermarket

and the time of the survey. If the findings are to be applied more widely, this is likely to involve making one or more assumptions. In this situation, any assumptions should be identified and justified.

AS 90647: *Use a mathematical model involving curve fitting to solve a problem*

For the award of Merit or Excellence for this standard, candidates must collect their own data. If a task is for Merit or Excellence only, and the candidate does not gain Merit, it is unlikely to provide evidence for Achievement. Comments at Excellence must relate to the analysis of the model, not the experiment. The model must be used to solve a problem. Justifications must be non-trivial.

Unit Standards

US 5244 does not permit the use of calculators.