3

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Statistics and Modelling, 2006

90643 Solve straightforward problems involving probability

Credits: Four
2.00 pm Tuesday 21 November 2006

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–STATF.

You should answer ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	Achievement Criteria	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Solve straightforward problems involving probability. ✓	Solve probability problems. —	Apply probability theory. —
	Overall Level of Performance	A

T2/2

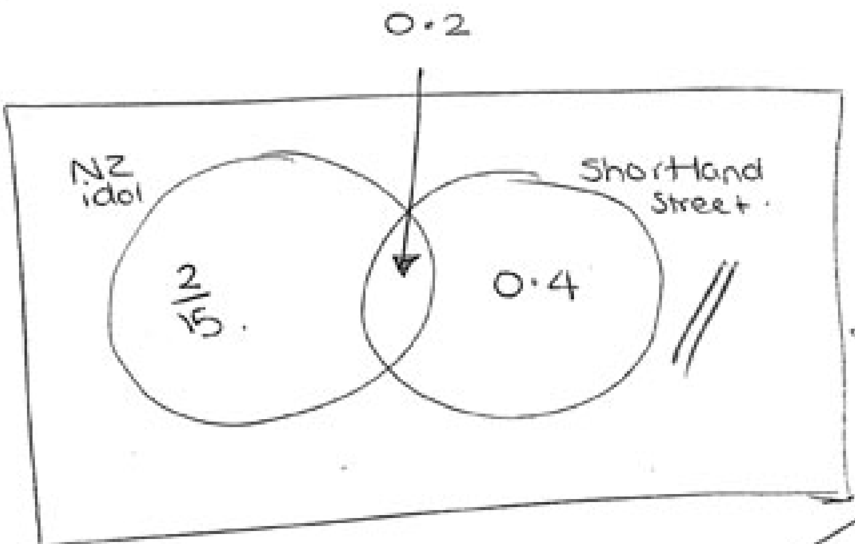
You are advised to spend 45 minutes answering the questions in this booklet.

Assessor's use only

QUESTION ONE

Rewa asked 150 randomly chosen students what programmes they had watched the previous night on television. *Shortland Street* was watched by 90 students, 50 students had watched *NZ Idol*, and 30 had watched both.

What is the probability that a randomly chosen student had watched neither *Shortland Street* nor *NZ Idol* the previous night?



P (student didn't watch Shortland Street or NZ idol) = ~~(2/15 + 0.2 + 0.4)~~

1 − (2/15 + 0.2 + 0.4)

= 0.2667

A

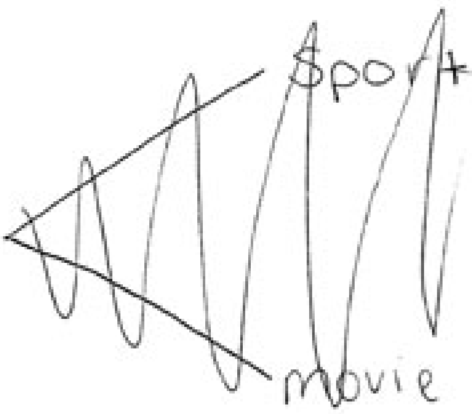
T2/3

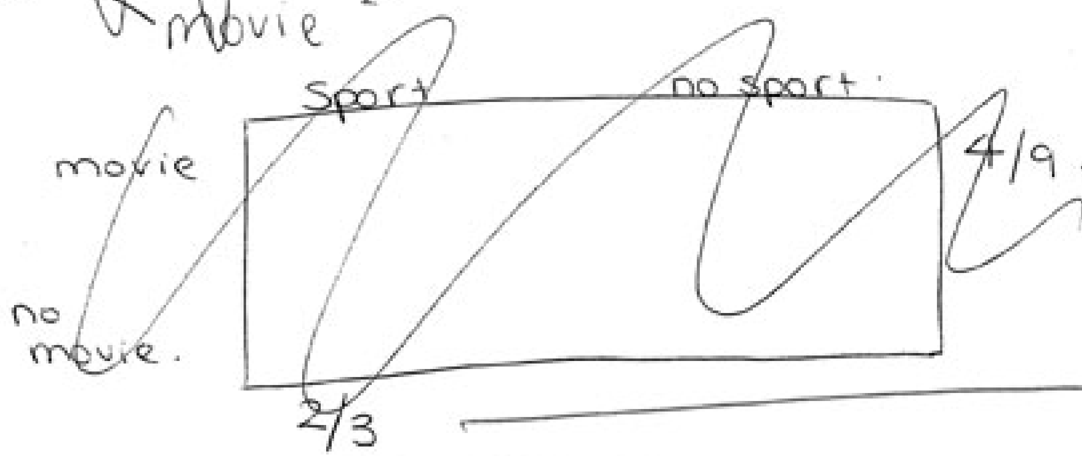
QUESTION TWO

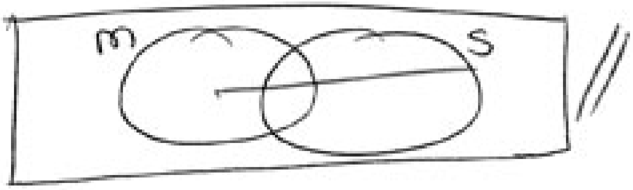
Assessor's use only

Stefan surveyed a different group of randomly chosen students about what types of television programmes they had watched over the weekend. He found that $\frac{2}{3}$ of them had watched sport, and that $\frac{4}{9}$ had watched a movie.

If $\frac{4}{5}$ of them had watched at least one of sport or movies, what is the probability that a randomly chosen student watched **both** sport and movies?







$P(S \cup M) = P(S) + P(M) - P(S \cap M)$

$4/5 = \frac{2}{3} + \frac{4}{9} - P(S \cap M)$

$P(\text{sport} \;\&\; \text{movie}) = 14/45 = 0.3111$

A

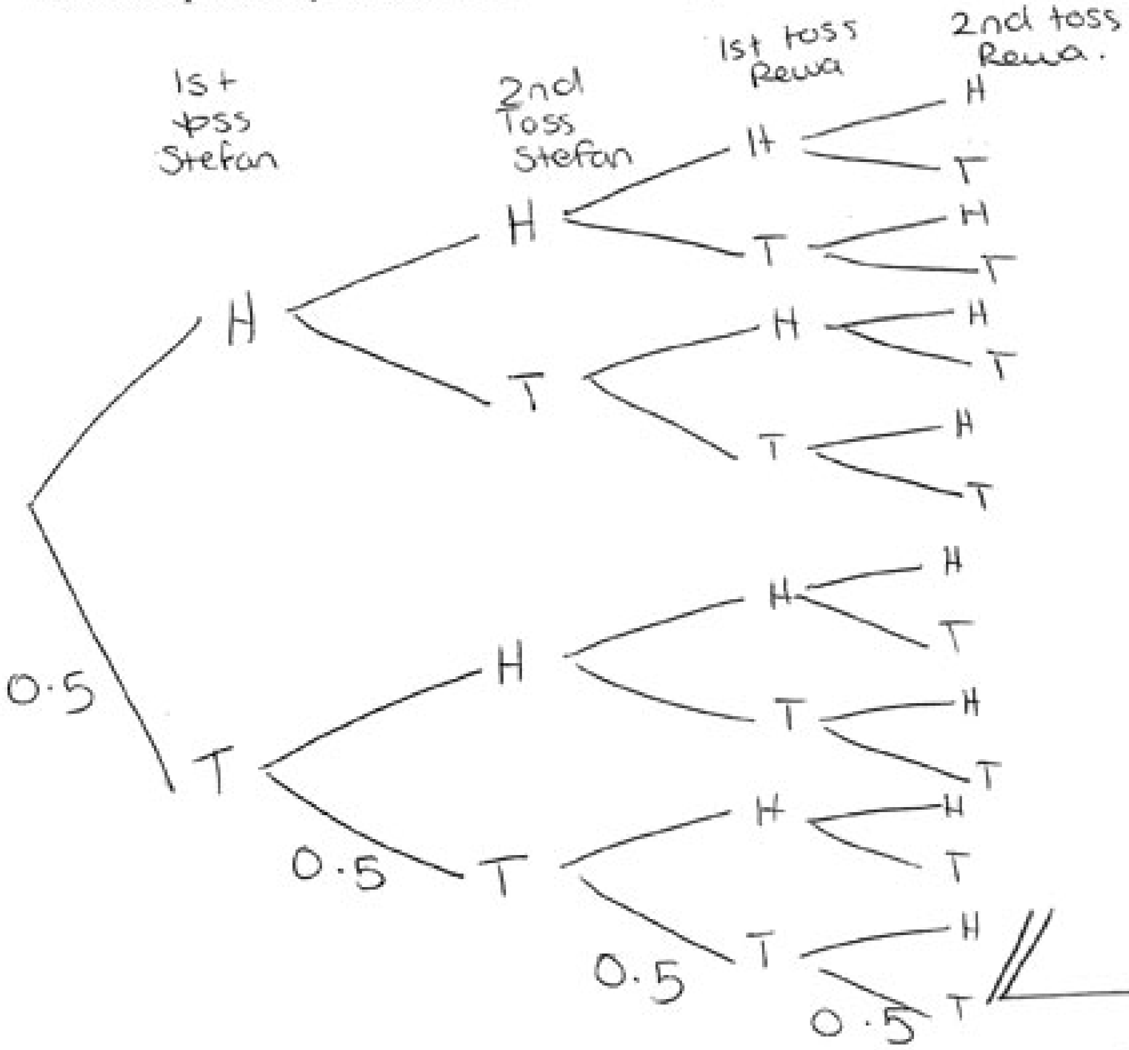
T2/4

Assessor's use only

QUESTION THREE

Stefan tosses two coins and Rewa also tosses two coins.

What is the probability that **both Stefan and Rewa get at least one head?**



P(~~to~~ get at least one head) = 1 − P(no heads)
= 1 − 0.0625
= 0.9375

NS

T2/5

QUESTION FOUR

Assessor's use only

Stefan and Rewa have one die each. Each face of each die has the same chance of landing face up. Stefan's die has three faces numbered 1, two faces numbered 2, and one face numbered 3. Rewa's die has two faces numbered 1, one face numbered 2, and three faces numbered 3.

Rewa and Stefan toss their dice together and record the total. What is the **expected total** of the two dice?

x	2	3	4	5	6
$P(X=x)$	$\left(\frac{1}{2}\times\frac{2}{6}\right) = 1/6$	$\left(\frac{3}{6}\times\frac{1}{6}\right)+\left(\frac{2}{6}\times\frac{2}{6}\right) = 7/36$	$\left(\frac{3}{6}\times\frac{3}{6}\right)+\left(\frac{2}{6}\times\frac{1}{6}\right)+\left(\frac{2}{6}\times\frac{1}{6}\right) = 13/36$	$\left(\frac{2}{6}\times\frac{3}{6}\right)+\left(\frac{1}{6}\times\frac{1}{6}\right) = 7/36$	$\frac{1}{6}\times\frac{3}{6} = \frac{1}{12}$

$$E(x) = \left(2\times\frac{1}{6}\right)+\left(3\times\frac{7}{36}\right)+\left(4\times\frac{13}{36}\right)+\left(5\times 7/36\right)+\left(6\times\frac{1}{12}\right)$$

$$E(x) = \frac{1}{3}+\frac{7}{12}+\frac{13}{9}+\frac{35}{36}+\frac{1}{2}$$

$= 3.833$

expected total (of two dice) $= 4$ #

A

T2/6

Assessor's use only

QUESTION FIVE

Rewa gets her car checked before she sells it. The probability that the car will need an oil change is 0.3, and the probability that the car needs a new oil filter is 0.5. The probability that both the oil and filter need changing is 0.225.

If the oil has to be changed, what is the probability that a new oil filter is needed?

$$P(\text{new oil filter} / \text{oil needs changing}) = \frac{P(\text{new oil filter} \cap \text{oil needs changing})}{P(\text{oil needs changing})}.$$

$$P(\text{new oil filter} / \text{oil needs changing}) = \frac{0.225}{0.3} = 0.75$$

M

QUESTION SIX

Rewa and Stefan are both members of the Student Council. The Student Council comprises ten members. When the Student Council is introduced to the school at an assembly, the Student Council members sit in a row on stage. The seats are allocated at random.

What is the probability that Rewa will be seated on the extreme left of the row, and Stefan will be seated on the extreme right of the row at the school assembly?

T2/7

Assessor's use only

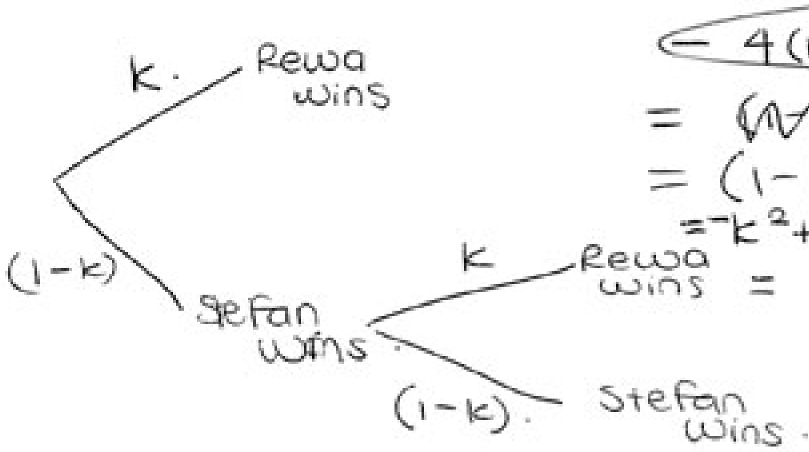
QUESTION SEVEN

Rewa and Stefan have been playing a game of chance. The game involves several rounds. The winner of each round gains one point. The overall winner of the game is the first person to gain a total of three points. The probability that Rewa wins any particular round is k.

Suppose that in the course of a game, Rewa has two points and Stefan has one point.
Let the random variable X be the number of rounds it takes for the game to be completed (ie the number of rounds before either Rewa or Stefan reaches three points, given that Rewa already has two points and Stefan has one point).

Show that $\mathrm{Var}(X) = k(1 - k)$

round 1



$$\mathrm{Var}(X) = (4(1-k)(1-k) + 4k(1-k) + k) - (2(1-k)(1-k) + 2k(1-k) + k)^2$$

$$= 4(1-k)^2 + 4k(1-k) + k$$

$$- 4(1-k)^2 + 4k(1-k)^2 + k^2$$

$$= (1-k)^2 + (1-k) + k$$

$$= -k^2 + 1 + 1 - k + k$$

$$= k(1-k)$$

$(1-k)(1-k)$

p(takes 1 more round) = k.

p(takes 2 more rounds) = $(1-k)^2 + k(1-k)$.

x^2	1	4
x	1	2
$P(X=x)$	k	$(1-k)^2 + k(1-k)$

$$E(X) = k + 2((1-k)^2 + k(1-k))$$

$$= 2(1-k)(1-k) + 2k(1-k) + k$$

$$E(X^2) = 4(1-k)(1-k) + 4k(1-k) + k$$

$\mathrm{Var}(X) =$

N5

T 2/8

Extra paper for continuation of answers if required.
Clearly number the question.

Assessor's use only

Question number