Foundation Examinations Spring 2004

March 06, 2004

QUANTITATIVE METHODS
(MARKS 100)
Module A
Q. 1 (a) The population of a city was 8 million on January 1, 2000. The population is growing at the exponential rate of 2 percent per year. What will the population be on January 1, 2005?
(b) A person borrowed Rs. 20,000 from a bank at a simple interest rate of 12 percent per annum. In how many years will he owe interest of Rs.3,600.
(c) The number of colour T.V. sets sold by a firm was three times the combined sale of C.D players and radios. If the sales included 72 T.V. sets and 8 radios, how many C.D. players were sold.
(d) Evaluate $\frac{d y}{d x} \quad$ at $x=1$, if
$y=\frac{U}{U+1}$ and $U=3 x^{2}-1$
(e)

Given, $\quad A=\left[\begin{array}{c}5 \\ 1 \\ 10\end{array}\right], \quad B=\left[\begin{array}{lll}3 & 9 & 4 \\ 2 & 1 & 8 \\ 5 & 6 & 1\end{array}\right]$
find, AB and BA , if possible.
Q. 2 (a) The arithmetic mean of profit earned by two companies ' X ' and ' Y ' is Rs. 34 million whereas the geometric mean is Rs. 16 million. Find out the profit earned by each company. It is known that company ' X ' earned more than ' Y '.
(b) How much money must be invested in an account at the end of each quarter if the objective is to have Rs. 225,000 after 10 years. The account can earn an interest rate of 9 percent per year compounded quarterly. How much interest will be earned over the period.
Q. 3 (a) Suppose the total cost of manufacturing ' $q$ ' units of a certain commodity is $c(q)=3 q^{2}+q+48$
(i) At what level of production is the average cost per unit the minimum?
(ii) At what level of production is the average cost per unit equal to the marginal cost?
(b) $\mathrm{M} / \mathrm{s}$ ABC Technologies know that the relationship between their weekly sales Q and weekly profit PR is expressed by the following function:

$$
\begin{equation*}
P R=-0.002 Q^{2}+10 Q-4000 \tag{04}
\end{equation*}
$$

Advise the company about the profit maximizing quantity.
Q. 4 (a) Graph the feasibility region for the following system of inequalities

$$
\begin{align*}
& 2 x-y=5 \\
& 3 x+y>0 \\
& x<4 \tag{06}
\end{align*}
$$

Find all corner points of the feasibility region.
(b) A firm produces three products $\mathrm{X}, \mathrm{Y}$ and Z with a profit of Rs. 20, Rs. 18 and Rs. 16 respectively. Production data are as follows:

|  | Machine <br> Hours | Labour <br> Hours | Raw Material <br> Units |
| :---: | :---: | :---: | :---: |
| X | 5 | 2 | 8 |
| Y | 3 | 5 | 10 |
| Z | 6 | 3 | 3 |
| Availability | 3,000 | 2,500 | 10,000 |

Set up the initial Simplex Tableau including the necessary slack variables.
Q. 5 (a) The following frequency distribution gives the weight of 35 objects, measured to the nearest kg. Draw a histogram to illustrate the data:

| Weight (Kg) | $6-8$ | $9-11$ | $12-17$ | $18-20$ | $21-29$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency | 4 | 6 | 10 | 3 | 12 |

(b) Following data relates to per unit prices of housing utilities i.e. Electricity, Gas, Water and Telephone:

| Utilities | Years |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | 2001 |  | 2002 |  |
|  | price | quantity | price | quantity |
| Electricity | 1.97 | 62 | 2.10 | 68 |
| Gas | 7.90 | 8.7 | 8.60 | 9.5 |
| Water | 0.29 | 296 | 0.31 | 298 |
| Telephone | 2.40 | 55 | 2.50 | 58 |

Calculate Fisher's index for the year 2002, using $2001=100$. Find the percentage increase in the year 2002 as compared to 2001.
Q. 6 (a) The following table shows the data on shelf space allotment ( x ) and sales ( y )

| Space allotted <br> (sq. feet-x) | Sales <br> (Number of boxes-y) |
| :---: | :---: |
| 2 | 20 |
| 4 | 36 |
| 6 | 38 |
| 8 | 38 |
| 10 | 52 |
| 12 | 54 |

(i) Determine the equation of least square regression line of y on x .
(ii) Using the above equation, estimate sales for a space allotment of 7 square feet.
(b) A computer while calculating the correlation co-efficient between two variables X and Y from 25 pairs of observation obtained the following sums:

$$
\begin{aligned}
& \mathrm{SX}=125 \\
& S X^{2}=650 \\
& S Y=100 \\
& S Y^{2}=460 \\
& S X Y=508
\end{aligned}
$$

The following mistakes were discovered at the time of checking:

| Wrong Values <br> Recorded |  | Correct Values Need to be <br> Recorded |  |
| :---: | :---: | :---: | :---: |
| X | Y | X | Y |
| 6 | 14 | 8 | 12 |
| 8 | 6 | 6 | 8 |

Find out the correct value of the co-efficient of correlation.
Q. 7 (a) The following table provides a relative frequency distribution for the size of the farms in the province of Punjab.

| Size (acres) |  | Relative <br> Frequency |
| :---: | :---: | :---: |
| Under 10 |  | 0.087 |
| $10-49$ |  | 0.192 |
| $50-99$ |  | 0.156 |
| $100-179$ |  | 0.173 |
| $180-259$ |  | 0.098 |
| $260-499$ |  | 0.143 |
| $500-999$ |  | 0.085 |
| $1000-1999$ | 0.040 |  |
| 2000 \& over | 0.026 |  |

A farm is selected at random, determine the probability that the farm selected has:
(i) less than 2000 acres
(ii) at least 50 acres
(b) In an introductory statistics class, the number of males and females are shown in the following frequency distribution table:

| Sex | Frequency |
| :--- | :---: |
| Male |  |
| Female | 23 |

Two students are selected at random without replacement from the class. Find the probability that the first student selected is female and second is male.
(c) Current medical studies show that 30 percent of the population will suffer from the common cold each winter. A group of 12 people is randomly selected.
i. What is the probability that exactly 5 in the group will have the common cold this winter?
ii. What is the probability that at least 5 in the group will have the common cold this winter?
iii. Compute the mean and variance of the number in the group that will have the common cold this winter.
Q. 8 (a) A physician wishes to estimate with $95 \%$ confidence the mean 'Serum Cholesterol' level of a population. He wishes the estimate to be within 5 units of the true mean. From previous work, he has learnt that the appropriate value of $\sigma=20$.

How large a sample he should take?
(b) The environmental protection officer of a large industrial plant sought to determine the mean daily amount of sulphur oxide emitted by the plant. A random sample of 10 days' measurements gave a mean of 9.5 tons per day with standard deviation of 3.24 tons per day. Suppose emissions per day are normally distributed.

Estimate ' $\mu$ ' using a $95 \%$ confidence interval and interpret your result.
Q. 9 For the following distribution, calculate the variance, standard deviation and coefficient of variation:

| $\mathbf{x}$ |  | $\mathbf{f}$ |
| :---: | :---: | :---: |
| $20-29$ |  | 4 |
| $30-39$ |  | 12 |
| $40-49$ |  | 19 |
| $50-59$ |  | 25 |
| $60-69$ |  | 20 |
| $70-79$ |  | 14 |
| $80-89$ |  | 6 |

