

THE BRITISH COMPUTER SOCIETY

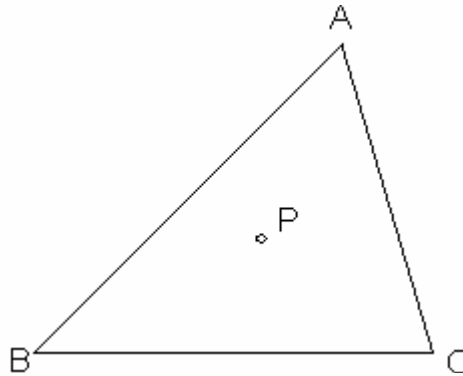
THE BCS PROFESSIONAL EXAMINATION 2002
Professional Graduate Diploma

COMPUTER GRAPHICS

SOLUTIONS

(This paper will be examined for the last time in April 2003)

1. (a) *Gouraud shading is a commonly used method for rendering polygon meshes.*
- (i) *What is the main improvement offered by Gouraud shading over constant intensity (or flat) shading?* (2 marks)
- (ii) *How can the normal vector at a vertex be determined from the surface normals of the polygons sharing that vertex?* (3 marks)
- (b) *Consider the following triangle which is to be colour-filled using Gouraud shading.*



- (i) *Given intensity values I_A , I_B and I_C for the three vertices A, B and C respectively, derive a formula for the intensity at the point P. [You may assume you are working with the intensities of just one colour – equations for the other colours will be analogous.]* (10 marks)
- (ii) *Derive an equation for the incremental interpolation of intensity along the edge AB.* (10 marks)

Answer Pointers

This question is designed to test the candidates' knowledge and understanding of Gouraud shading.

- (a) (i) For **curved surfaces** which have been approximated by polygon meshes **flat shading will highlight the polygon edges whilst Gouraud shading will smooth the edges out.** 1 mark for each bolded point
- (ii) The normal vector at a vertex is taken **as the average of the surface normals of the polygons sharing the vertex.**

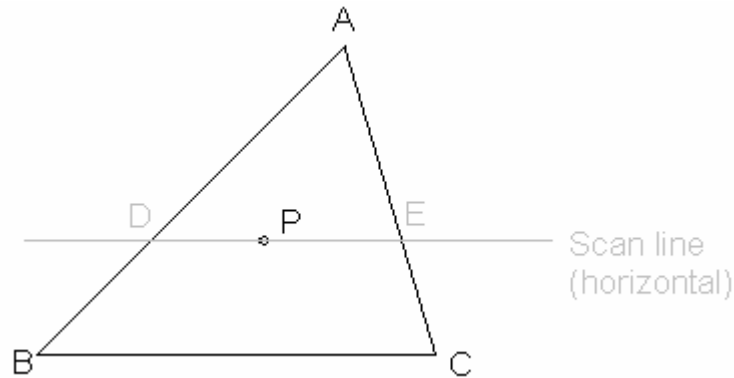
$$N_v = \frac{\sum_k N_k}{|\sum_k N_k|}$$

where N_v is the normal at the vertex
 N_k are the surface normals of the polygons

i.e. the **sum of the polygon normals divided by their combined magnitude.**

1 mark for each textual description and 1 mark for the equation

(b) (i)



We consider the scan line running through P and derive intermediate intensities along AB and AC at the points D and E. We need only consider the vertical displacements of A, B and C because we are dealing with straight lines –

$$I_D = \frac{y_D - y_B}{y_A - y_B} I_A + \frac{y_A - y_D}{y_A - y_B} I_B$$

I_E is determined in a similar fashion.

Now we can determine the intensity at P using I_D and I_E and the horizontal displacements of D, E and P –

$$I_P = \frac{x_E - x_P}{x_E - x_D} I_D + \frac{x_P - x_D}{x_E - x_D} I_E$$

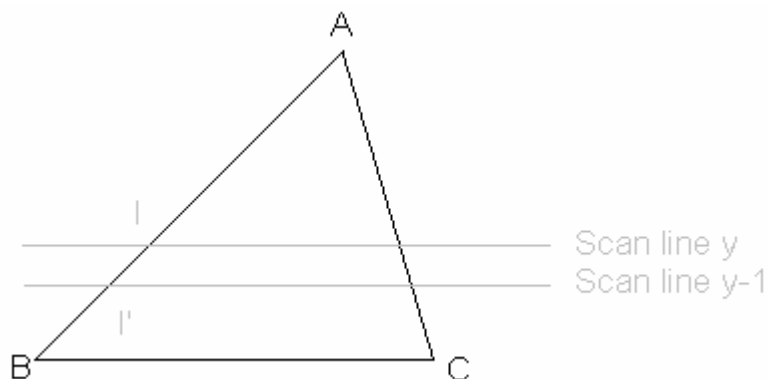
1 mark for scan line diagram

1 mark for similarity of D and E

4 marks for each equation – total 8 marks

(ii)

1 mark for diagram



The intensity, I , at a position (x,y) on the edge AB can be interpolated as

$$I = \frac{y - y_B}{y_A - y_B} I_A + \frac{y_A - y}{y_A - y_B} I_B$$

4 marks for equation

We can obtain the intensity, I' , at the next scan line, $y+1$, along AB from

$$I' = I + \frac{I_B - I_A}{y_A - y_B}$$

4 marks for equation

Similar calculations allow us to obtain the intensities at successive horizontal positions along the scan line.

1 mark for horizontal aspect

2. (a) *Texture mapping is frequently used to improve visual realism.*
- (i) *What are the three co-ordinate spaces used in texture mapping and how do they relate to each other?* (6 marks)
- (ii) *Why is inverse scanning preferred in texture mapping?*
[You should make reference to pixel pre-images and anti-aliasing in your answer.] (5 marks)
- (iii) *What further improvement to realism does bump mapping introduce?* (4 marks)
- (b) *Consider the problem of mapping a texture defined on the unit square onto the surface of a cylinder of radius r and height h .*
- (i) *Develop a suitable tiling scheme for this task.* (5 marks)
- (ii) *Thence derive the mapping equations.* (5 marks)
[You may ignore viewing and projection transformations as well as aliasing problems here.]

Answer Pointers

This question is designed to test the candidates' knowledge and understanding of texture mapping.

- (a) (i) The three co-ordinate spaces are –

Image Space
 Uses pixel co-ordinates (x,y)
 Object Space
 Uses surface parameters (u,v)
 Texture Space
 Uses array co-ordinates (s,t)

1 mark for each space

The Image and Object spaces are related by **the usual viewing and projection transformations**.

1 mark

The Object and Texture spaces are related by a **user defined surface parameterisation** which “glues” the texture onto the object surface.

2 marks

- (ii) Inverse scanning is preferred because **texture patches are unlikely to match nicely with pixel boundaries**. By working backwards from pixels to texture patterns **pixel pre-images can be identified within the texture pattern** and **subsequently processed to obtain the “best” value for that pixel**. Furthermore **aliasing becomes a big problem if textures are used** and the inverse scanning approach facilitates the **introduction of anti-aliasing techniques into the mapping procedure**.

1 mark for each bolded point

5 total

- (iii) Bump mapping applies **perturbations** to the **surface normals** and thus **creates the impression of surface roughness characteristics** when a **local illumination model** is applied.

1 mark for each bolded point

4 total

- (b) (i) A scheme which maps points on a rectangle of size $2 \times \pi r$ by h onto the unit square is required.

Let (s, t) be the texture space co-ordinates and (u, v) be the object space co-ordinates (u is the displacement around the circumference and v is the vertical displacement).

$$0 \leq t \leq 1 \text{ and } 0 \leq v \leq h$$

Set $t = v - \text{trunc}(v)$ to map the vertical displacements

$$0 \leq s \leq 1 \text{ and } 0 \leq u \leq 2 \times \pi r$$

Set $s = u - \text{trunc}(u)$ to wrap the texture around the cylinder

5 marks for a mapping whose domain is the whole surface

- (ii) To map from image space to object space (ignoring viewing and projection transformations) a mapping from (x, y, z) onto (u, v) is required.

Set $v = z$ for $0 \leq z \leq h$ and undefined elsewhere

Set $u = (\text{atan2}(y, x) + \pi)r$ since the range of atan2 is $-\pi \rightarrow \pi$

We can use these (u, v) values in the tiling scheme specified in (i) above and thus, for all (x, y, z) on the surface of the cylinder, we can determine an (s, t) to obtain the texture for that pixel from the texture pattern.

5 marks for relating image through to texture like this

3. (a) Describe the functionality of a Window Manager in an X Windows implementation.

(10 marks)

- (b) Describe the control functions which a Window Manager typically adds to each X client application, giving examples of the user interface for each control.

(15 marks)

Answer Pointers

- (a) A Window Manager is a program implemented with Xlib to control the layout of windows on the screen, responding to user requests to move, resize, raise, lower or iconify windows. The Window Manager may also enforce a policy, such as mandatory window or icon size and placement, and provide a menu for commonly used functions. 10 marks
- (b) Functions:
- move - allow the user to relocate a window - active areas on window borders 3 marks
 - resize - allow the user to resize a window - active areas on window corners 3 marks
 - raise/lower - allow the user to cover or uncover a window with another - menu on window bar 3 marks
 - iconify - allow the user to reduce a window to an icon - button on window bar 3 marks
 - focus - allow the user to move keyboard focus into a window - clicking in window or moving cursor into window 3 marks
4. A drawing package needs to be able to define and to display Hermite curves. Your answers to each section should include a diagram where appropriate.
- (a) Describe the controls which are required to generate a Hermite curve. (4 marks)
- (b) Show how a family of Hermite curves can be generated by changing the value of one of the controls. (3 marks)
- (c) Give the necessary conditions for two Hermite splines to be combined with G0 continuity. (4 marks)
- (d) Give the necessary conditions for two Hermite splines to be combined with G1 continuity. (4 marks)
- (e) Give the necessary conditions for two Hermite splines to be combined with C1 continuity. (4 marks)
- (f) Describe Horner's rule for reducing the number of calculations when evaluating a cubic polynomial, giving the number of calculations required both with and without the use of Horner's rule. (6 marks)

Answer Pointers

- (a) A Hermite curve is defined by its two end points and the tangent vectors at the two end points. 4 marks
- (b) A family of curves can be created by rotating (or changing the magnitude of) the tangent vector at one of its end points. 3 marks
- (c) Two Hermite curves combine to make a G0 continuous curve by making the two adjacent sections have a common end point. 4 marks
- (d) Two Hermite curves combine to make a G1 continuous curve by making the two adjacent sections have a common end point and the common tangent vectors must have the same direction.

4 marks

- (e) Two Hermite curves combine to make a C1 continuous curve by making the two adjacent sections have a common end point and the common tangent vector must have the same direction and magnitude.

4 marks

- (f) at^3+bt^2+ct+d requires 6 multiplications and 3 additions. Horner's rule $((at+b)t+c)t+d$ requires 3 multiplications and 3 additions.

6 marks

5. A drawing package uses scan conversion algorithms to display a number of primitive shapes. The implementation of the algorithms must be computationally efficient. The shapes must not have any visual irregularities. Your answers to each section should include a diagram where appropriate.

- (a) A quarter circle of radius R is scan converted by incrementing the x ordinate in unit steps and calculating and rounding the y ordinate. Explain why this scan conversion approach is unacceptable.

(8 marks)

- (b) Two straight lines are drawn using a scan conversion algorithm. One has a gradient of 0, the other has a gradient of 1. Both lines are drawn using 20 pixels. Explain why the two lines appear to have different intensities and provide TWO mechanisms by which this effect can be eliminated.

(9 marks)

- (c) Given that the pixel at co-ordinates (x, y) lies on a circle centred at $(0,0)$, using considerations of symmetry state which other pixel co-ordinate points are known to lie on the circle. Which values of x and y , which lie on the circle, cannot have their additional points generated in this way? Give the reason why and state the additional points that can be generated from each of these values.

(8 marks)

Answer Pointers

- (a) The computation requires a square and a square root computation for each point which is computationally expensive. The resulting circle has gaps as the y ordinates get further apart as x approaches R .

8 marks

- (b) The line of gradient 1 is longer than the horizontal line by a factor of root 2, hence the longer line has an overall intensity of 71% of the shorter line.

4 marks

This effect can be eliminated by modifying the pixel intensity according to the gradient of the line, giving the horizontal line a pixel intensity of 71% of the gradient one line gives equal overall intensity.

3 marks

The effect can also be eliminated by treating the lines as rectangles and applying anti-aliasing techniques.

2 marks

- (c) The point (x, y) can be reflected to form the points $(x, -y)$, $(-x, y)$, $(-x, -y)$, (y, x) , $(y, -x)$, $(-y, x)$ and $(-y, -x)$.

3 marks

The above cannot be applied if $y = x$, if $x = 0$, or if $y = 0$ as each pixel will be drawn twice which can cause display anomalies, particularly if using an XOR drawing mode.

2 marks

If $x = y$, then (x, x) can be reflected to form $(x, -x)$, $(-x, x)$, $(-x, -x)$.

1 mark

If $x = 0$, then $(0, y)$ can be reflected to form $(0, -y)$, $(y, 0)$, $(-y, 0)$.

If $y = 0$, then $(x, 0)$ can be reflected to form $(-x, 0)$, $(0, x)$, $(0, -x)$.

1 mark

1 mark