UNIVERSITY OF KWAZULU-NATAL
EXAMINATIONS: 13 JUNE 2005

SUBJECT, COURSE AND CODE: MATHEMATICS 001 (MATH001)

DURATION: 3 HOURS **MAXIMUM MARKS: 150**

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PLEASE NOTE: This question paper consists of 6 pages.
Please see that you have them all.

Answer all questions and show all working details.

Question 1

Complete each of the following statements in your answer book. **Do not re-write the statements.**

a) The sum of two odd numbers is always an ..... number.

b) An integer divided by a non-zero integer always yields a ..... number.

c) If k is a natural number then $7k$ is a ..... of 7.

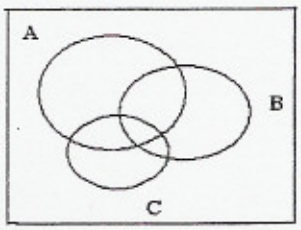
d) If P is the set of prime numbers and K is the set of composite numbers then $P \cap K =$ .....

e) A straight line graph through the origin is a characteristic of ..... proportion.

[5]

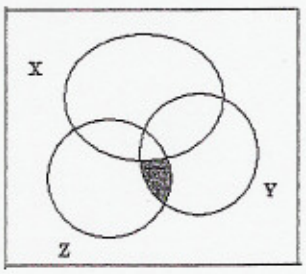
Question 2

a) Copy the diagram below in your answer book and shade $(A \cup B)' \cap C$.



(3)

b) Describe, mathematically, the shaded region below.



(3,5)

[6,5]

Question 3

a) Consider the universal set, $E = \{\sqrt{17}\ ;\ 2\pi\ ;\ \sqrt{36}\ ;\ 3\ ;\ \sqrt{-7}\ ;\ 2{,}6\dot{4}\}$,
A = {prime numbers in E}, B = {real numbers in E}
and C = {irrational numbers in E}.
i) Draw a Venn diagram of E, A, B and C, writing all the elements in the correct regions. (4,5)
ii) List the elements of $C \cup A$. (1)
iii) Determine $n(A' \cap B)$. (1)
iv) How are C and B related? (1)

b) Consider the sets $P = \{5;3k\}$ and $Q = \{5k\}$, where k is a real constant.
If P and Q are disjoint sets then determine the value(s) k can not have. (3)

[10,5]

Question 4

a) Prove $2{,}4\dot{1}$ is a rational number. (4,5)

b) Prove that the sum of three consecutive natural numbers is always divisible by 3. (4)

c) "If n is a natural number then $(2^n - 1)$ is always a prime number."
Disprove this statement. (3)

d) Observe the following pattern

$$1^3 = 1^2 - 0^2$$
$$2^3 = 3^2 - 1^2$$
$$3^3 = 6^2 - 3^2$$
$$4^3 = 10^2 - 6^2$$

Using the above pattern, determine $a + b$ if $10^3 = a^2 - b^2$. (4)

[15,5]

Question 5

A cylindrical container, with radius equal to 10 m, is completely filled with 750 m^3 of water. At 11h00, the container of water starts to leak. The water leaks from the container at a constant rate of 4 m^3/minute. Determine the height of the water in the container at 11h20. **(Write your answer correct to 2 significant figures.)**

[8]

Question 6

The strength, s, of a wooden plank is given by the formula

$$s = \frac{kw^2 l}{t}$$

where k is a positive constant, w is the width of the plank , l is the length of the plank and t is the thickness of the plank. All measurements are in metres.

Consider the following information regarding the dimensions of two planks.

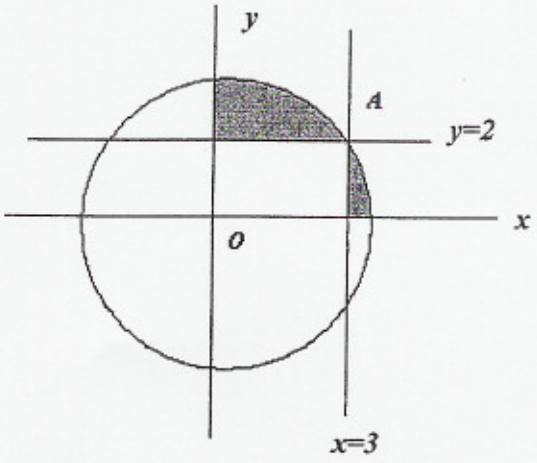
Plank 1	**Plank 2**
* 8 metres thick	* 25% thicker than Plank 1
* length is x metres	* twice as long as Plank 1
* width is $3y$ metres	* half as wide as Plank 1

Based on the above dimensions of each plank, decide, **showing all calculations**, which plank has the greater strength.

[9]

Question 7

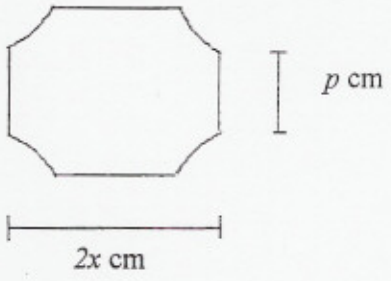
In the diagram below, the lines $y = 2$ and $x = 3$ intersect a circle, with centre at the origin O, at point A. Determine the area of the shaded region in terms of π.



[6]

Question 8

The diagram below represents the cross section of a right prism. Each of the curved lines represent quarter circles of radius $4p$ cm.



a) Determine the perimeter of the cross section in terms of π, p and x. (5)
b) Determine the area of the cross section in terms of π, p and x. (4)
c) If the prism is open at one end, and the height of the prism is p cm, determine the total surface area of the prism in terms of π, p and x.

(2,5)
[11,5]

Question 9

The **last stage** in a Gauss reduction calculation is shown below. Study the information carefully and answer the questions that follow.

x	y	z	1
a	$3b$	p	r
0	k	l	m
0	c	d	16

a) Write down the value(s) of c. (2)
b) Write down the value(s) of d for which
i) The above system of linear equations has no solution. (2)
ii) $z = 8$ is a root of the system of linear equations described above. (3)

[7]

Question 10

a) From the Gauss reduction table below, solve for x, y and z. (5)

x	y	z	1
2	1	-1	4
0	2	1	5
0	0	0	0

b) Solve for x
i) $2x < x + 4 \leq 3x - 1$ (Write your solution using set builder notation) (6)
ii) $\frac{3}{x+1} \geq \frac{4}{x-2}$ (Express your solution in interval notation) (8)

c) On Jubalo's farm, the only animals are hens and goats. When Jubalo counted the heads and legs of the animals, he found that there are 19 heads and 66 legs altogether. Determine the number of hens and goats Jubalo has on his farm. (6)

d) Eric left for Johannesburg by car at 6:00 and travelled at an average speed of 100 km/hr. His neighbour , John left for Johannesburg at 7:00 but travelled at average speed of 120 km/hr. If Eric stopped at 7:00 for 15 min to fill up petrol, determine at what time John will catch up with Eric? (7)

[32]

Question 11

a) Consider $p(x) = x^2 - 9$ and $q(x) = x + 3$.

i) Evaluate $(p \circ q)(2)$. (2)

ii) Write down the range of $p(x)$ using interval notation. (2)

iii) Determine the equation of the inverse of $p(x)$ in the form $y = \ldots$. (3)

iv) Explain briefly why the inverse of $p(x)$ is NOT a function. (1)

v) Suppose $t(x) = x^2 - 9, x > 0$. Write down the equation of the inverse of $t(x)$ in the form $y = \ldots$. (1)

vi) Sketch $\{(x;y) : y \geq x^2 - 9, x \in R\} \cap \{(x;y) : y \leq x + 3, x \in R\}$ (7)

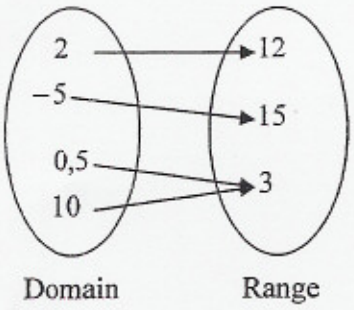
(Clearly label all intercepts and graphs)

b) If $f(x) = x + 1$ and $g(x) = \dfrac{4}{x-2}$

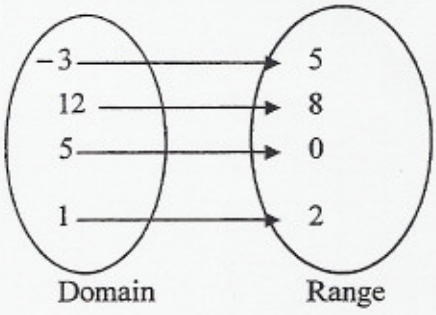
i) Show that $(g \circ f)(x) = \dfrac{4}{x-1}$. (1)

ii) Sketch $(g \circ f)(x)$. (5)

c) Carefully study each of the mapping diagrams below and **use the mapping diagrams to answer** the questions which follow.



Mapping diagram of $f(x)$



Mapping diagram of $g(x)$

Evaluate i) $3f(0,5)$ (1)

ii) $g^{-1}(0)$ (1)

iii) $(f \circ g)(1)$ (1)

iv) $(g \circ g)(-3)$ (2)

v) $(g \circ f)(-5)$ (2)

[29]

Question 12

a) f is a straight line graph with the following properties :

- $f(0) = 10$
- $f(x) \geq 0$ when $x \geq -2$.

Determine the equation of f. (4)

b) A parabola, g, has range $[2 ; \infty)$. The parabola is symmetrical about the line $x = 1$. The parabola cuts the y-axis at 3. Determine the equation of the parabola in the form $g(x) = ax^2 + bx + c$. (6)

[10]