## Spin-lattice relaxation, $T_{1}$

Time constant describing the approach of the magnetisation to thermal equilibrium that is the Boltzmann distribution

Exponential process:

$$
\frac{d M_{z}}{d t}=\frac{-\left(M_{z}-M_{0}\right)}{T_{1}}
$$

Determines

- initial polarisation in the pulse NMR experiment
- pulse repetition rate optimum sensitivity quantitative analysis

Requirement for relaxation
Fluctuating magnetic field at the Larmor frequency

Brownian motion of molecules in solution modulates the dipole-dipole coupling

## Effect of the correlation time

Distribution of frequencies associated with the fluctuating field.

- spectral density, $J(\omega)$

Correlation time, $\tau_{c}$, average time between molecular collisions

Spectral density changes with the correlation time

$$
J(\omega)=\frac{2}{5} \frac{\boldsymbol{\tau}_{c}}{1+\left(\omega \boldsymbol{\tau}_{c}\right)^{2}}
$$

Area under curve of $\mathrm{J}(\omega)$ against $\omega$ constant

## Consequences

- slow motion, fluctuating field does not generate significant components at Larmor frequency. $\boldsymbol{T}_{\mathbf{1}}$ long
$-\omega=\tau \mathbf{c}^{\mathbf{- 1}}$, maximise frequency component at Larmor frequency. $\boldsymbol{T}_{\mathbf{1}}$ short
- fast motion, spectral density more spread out decreasing component at the Larmor frequency. $T_{1}$ long


## Effect of the correlation time

Spin-lattice relaxation as a function of $\tau_{c}$

$$
\frac{1}{T_{1}}=\gamma^{2}\left[B_{x l}\right]^{2}(J(\omega)+J(2 \omega))
$$

## First term

- magnitude of the local fields generated by the molecular motion

Second term

- frequency distribution based on $J(\omega)$

Overall we see:

Variation of T1 with correlation time

$-\log (\mathrm{tau})$

## Measurement of the spin-lattice relaxation time

## 1. Inversion-recovery

A. Pulse sequence:
$\left[180_{\mathrm{x}}-\mathrm{t}-90_{\mathrm{x}}-(\mathrm{FID})-\operatorname{recycle} \operatorname{delay}\left(5 \mathrm{x} T_{1}\right)\right]$
B. Vector diagram description:

initial state
$\qquad$

population inverted
$\tau$

magnetisation
relaxes back to equilibrium

## Measurement of the spin-lattice relaxation time

## 1. Inversion-recovery <br> C. Data analysis

Magnetisation after a time, $\tau$, is given by:

$$
\mathrm{M}(\tau)=\mathrm{M}(\infty)\left[1-2 \exp \left(-\tau / T_{1}\right)\right]
$$

Typical set of data:


Estimate of $T_{1}$ from $\tau_{\text {null }}$

$$
T_{1}=\frac{\tau_{\text {null }}}{\ln 2}
$$

## Spin-spin relaxation time , $\boldsymbol{T}_{2}$

Time constant for the decay of the magnetisation components in the xy plane

Exponential process:

$$
\frac{d M_{x, y}}{d t}=\frac{-M_{x, y}}{T_{2}}
$$

N.B. At thermal equilibrium there are no magnetisation components in the xy plane

Determines

- the rate of decay of the FID
- the linewidth in the NMR spectrum for a lorentzian line

$$
v_{1 / 2}=1 / \pi T_{2}
$$

Two components

## 1. $T_{1}$ lifetime

magnetisation component in the xy plane must at least as fast as $T_{1}$.

Mechanism for relaxation the same as $T_{1}$, fluctuating field at the Larmor frequency

Implies a linewidth, $v_{1 / 2}>1 / T_{1}$
2. Nuclear spins exchange magnetisation

Flip-flop transitions. Magnetisation vectors in the xy plane fan out, loss of phase coherence.

Requires two coupled spins with the same transition energy. Energy conserving so occurs at zero frequency. No motion is necessary

Two limiting cases:

1. Extreme narrowing: $T_{1}=T_{2}$

Dominated by the first term
For molecules reorienting very rapidly;

$$
\omega_{\mathrm{o}} \tau_{\mathrm{c}} \ll 1
$$

2. In a rigid solid: $\quad T_{1} \gg T_{2} \sim 20 \mu \mathrm{~s}$

Dominated by the zero frequency term No motion

## Effect of magnetic inhomogeneity on $\boldsymbol{T}_{2}$

- Homogeneity of the applied field $B_{0}$ better than 1 part in $10^{10}$
- Small differences in $B_{0}, \delta B$, slightly alter the resonance condition

$$
\omega_{0}+\delta \omega=\gamma\left(B_{0}+\delta B\right)
$$



- Net magnetisation along x or y axis reduced, hence signal decays faster than the real $T_{2}$
- Effective $T_{2}$ in the presence of magnet inhomogeneity known as $T_{2}{ }^{*}$
- Linewidth in the NMR spectrum now

$$
v_{1 / 2}=1 / \pi T_{2} *
$$

## Spin echoes

- Refocus the dephasing caused by $\delta \mathrm{B}$
- No chemical shift evolution after the full period of the spin echo
- Basis of the Carr-Purcell-Meiboom-Gill (CPMG) pulse sequence for measuring $T_{2}$
- Pulse sequence

$$
90_{x}-\tau-180_{y}-\tau-\text { Echo }
$$

- Vector diagram

Spin echoes

dephasing with $\tau$



- Signal decays by $T_{2}$

