Math 0110 – Test 2

Implicit Differentiation

To differentiate a function where y isn't isolated (i.e. it's not in the form y = [*stuff involving x*]), we need to use implicit differentiation. The easiest way to understand implicit differentiation is to think of implicit differentiation as an extension of the Chain Rule**. Whenever we differentiate a term containing y, we must remember that the derivative of y is y'.**

Example:
Find the derivative of $x^2 + y^2 = 10$

Solution:
We can differentiate the x^2 term as we normally would. So, the first part of our derivative is just $2x$

To differentiate the y^2 term, think of it as $(y)^2$ and just differentiate it as you would using the Chain Rule.

We can start by taking care of the exponent: $2y$

Now, using the Chain Rule, we multiply by the derivative of the "inside". The inside is just y, so the derivative is y'. Putting it all together, the derivative of the left side is: $2x + 2y \times y'$. The derivative of the left side, a constant, is 0. So, we have:

$$2x + 2yy' = 0$$
$$2yy' = -2x$$
$$y' = -\frac{2x}{2y}$$
$$y' = -\frac{x}{y}$$

Example:
Find the derivative of $xy = 3x$

Solution:
On the left side, we're going to use the product rule. The derivative of x is 1 and the derivative of y is y'. Keeping this in mind, differentiating gives us:

$$xy' + y = 3$$

Now, we can solve for y':

$$xy' = 3 - y$$
$$y' = \frac{3-y}{x}$$

Related Rates

Related rates involve using knowledge about how one variable is changing over time to determine how another variable is changing over time.

Nearly all related rates problems require the use of a diagram.

To solve a related rates problem:

1. Draw a picture of the situation and assign variables.
2. Translate the word problem into mathematical equations.
3. Determine an equation relating all variables together
4. Differentiate that equation
5. Replace each variable and derivative in the differentiated equation with the value given in the problem. Solve for the required unknown.

Critical Values

The critical values of a function are found by solving where $f'(x) = 0$ and where $f'(x)$ does not exist.

A critical value is a value of x where the function can potentially change from increasing to decreasing or vice versa.

Relative Maxima and Minima

To find the maxima and minima of a function:

1. Find all critical points
2. Determine where the function is increasing and where it is decreasing (by looking at the sign of $f'(x)$)
3. If $f(x)$ switches from increasing to decreasing, the point is a maximum. If $f(x)$ switches from decreasing to increasing, the point is a minimum.

Alternative method:

1. Find all critical points
2. Find $f''(x)$ and evaluate $f''(x)$ at each critical point. If $f''(x) > 0$, then that critical point is a minimum. If $f''(x) < 0$, then that point is a maximum.

Inflection Points

Inflection points are analogous to critical values except instead of involving the first derivative, they involve the second derivative. To find inflection points, we need to determine when $f''(x) = 0$ and when $f''(x)$ does not exist.

Inflection points are x values where a function switches concavity.

Vertical Asymptotes

A function has a vertical asymptote whenever the denominator is equal to zero.

For example, the function $f(x)=\frac{x^2+4}{x-3}$ has a vertical asymptote when x – 3 = 0 or when x = 3.

Horizontal Asymptotes

If either $\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$ exist, then f(x) has a horizontal asymptote at that value.

For example, since $\lim_{x\to\infty}\frac{1}{x}=0$, the graph of y = 1/x has a horizontal asymptote at y = 0.