Math 0110 Study Sheet

Domain and Range

The domain of a function is the set of x values at which the functions "makes sense".

To find the domain of a function, you must check that:

1. anything inside a square root must be greater than or equal to zero
2. anything that appears in the denominator is never equal to zero

For example, the function $f(x)=\sqrt{x-1}$ will only make sense if $x - 1 \geq 0$ (you can't take the square root of a negative number). So, the domain is $x \geq 1$ or $[1, \infty)$

The function $f(x)=\frac{1}{x-3}$ won't make sense if x = 3 (you can't divide by zero). So, the domain is all real numbers except for x = 3.

The range is analogous to the domain. The range asks what y-values are produced by a function.

Composition of Functions

Composition is a way to take two functions and use them to form a new function.

$$f \circ g = f\left(g(x)\right)$$

For example, if

$$f(x)=x^2+x+1$$
$$g(x)=3\sqrt{x}$$

Then,

$$\begin{aligned}
&f \circ g \\
&= f\left(g(x)\right) \\
&= f\left(3\sqrt{x}\right) \\
&=\left(3\sqrt{x}\right)^2+\left(3\sqrt{x}\right)+1 \\
&=9x+3\sqrt{x}+1
\end{aligned}$$

Note: $f \circ g$ is almost never equal to $g \circ f$

Limits

A limit is a way of asking what a function is doing at a certain point if we weren't allowed to actually look and see what it was really doing at that point.

If the limit for a function is ∞ or $-\infty$, then we say the limit does not exist.

A few strategies for evaluating limits:

1. Plug in the value for x

   $$\lim_{x \to 3} \frac{1}{x} = \frac{1}{3}$$

   If there's nothing stopping you from evaluating the function at your point, then just plug in the number for x.

2. If you have a rational function and you can't just plug in the value for x, try factoring either the numerator or denominator (or both)

   $$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

   The numerator contains a difference of squares and can be factored

   $$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \lim_{x \to 2} x + 2 = 4$$

3. If the limit contains an absolute value, try to use the definition of the absolute value to rewrite it in simpler terms

   $$\lim_{x \to 0^+} \frac{|x|}{x}$$

   The definition of the absolute value of x is:

   $$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

   Since we're looking at when x approaches 0 from the positive direction, we really only care about when x > 0. So, we can replace |x| with x:

   $$\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1$$

Infinite Limits

Remember: $\frac{1}{\infty} = 0$

To evaluate the limit of a rational function as x approaches ∞ (or $-\infty$), divide the numerator and denominator by the highest degree term of x

For example:

$$\lim_{x\to\infty} \frac{3x^2 + x + 1}{2x^2 + 5}$$

$$= \lim_{x\to\infty} \frac{3x^2 + x + 1}{2x^2 + 5} \times \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{3x^2}{x^2} + \frac{x}{x^2} + \frac{1}{x^2}}{\frac{2x^2}{x^2} + \frac{5}{x^2}}$$

$$= \lim_{x\to\infty} \frac{3 + \frac{1}{x} + \frac{1}{x^2}}{2 + \frac{5}{x^2}}$$

$$= \frac{3}{2}$$

Velocity and Acceleration

If s(t) is a function representing the height (or displacement) of an object.
Then v(t), the velocity, is equal to the first derivative: s'(t)
And a(t), the acceleration, is equal to the second derivative: s''(t)

Common Questions

To find when the object will hit the ground, set s(t) = 0 and solve for t
To find the instantaneous velocity at 5 seconds, find s'(t) and plug in t = 5
To find the average velocity between two times (t_1 and t_2), evaluate:

$$\frac{s(t_1) - s(t_2)}{t_1 - t_2}$$

(This is just the slope formula)

Graphs

An open circle indicates that there is no point at the spot. In Diagram 1, there is no point at (2, 1)
A colored circle indicates that there *is* a point. For example, we have a point at (-1, 1)

To find where a function is discontinuous by examining its graph: find all the points where the line is broken. In the graph below, the function is discontinuous at x = 2 and x = -1

The graph only exists for values of x between -5 and 4, so the domain is [-5, 4]
The range of the graph will include all values between -2 and 3, or [-2, 3]

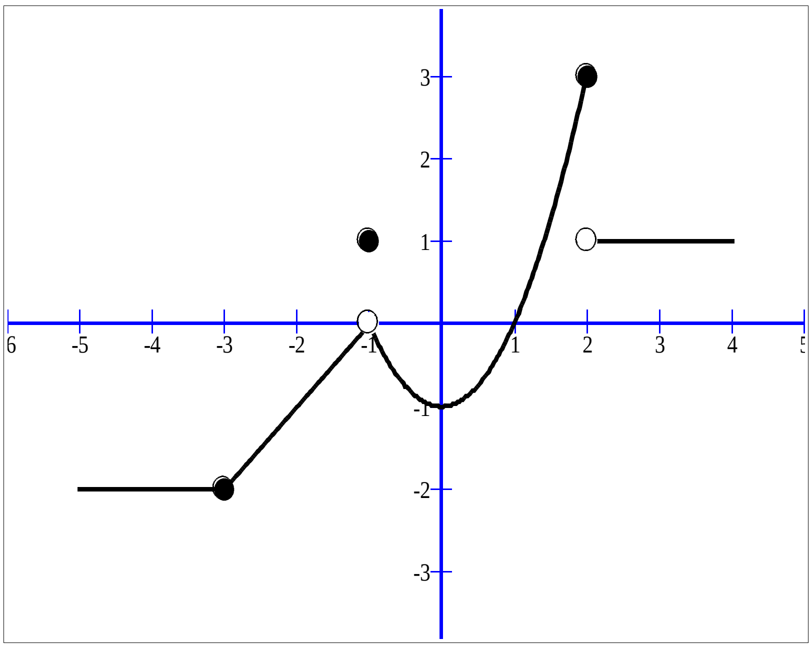


Diagram 1

$\lim_{x \to 2^+} f(x)$ asks what we'd expect the function to be doing at x = 2, if we weren't allowed to actually look at what it was really doing at that point. The + tells us that we're only considering what the graph appears to be doing as we approach x = 2 from the *right* (or positive) side. For the graph above, it appears that we'd be approaching 1 (it doesn't matter whether a point exists there or not).
So, $\lim_{x \to 2^+} f(x) = 1$.

Likewise, $\lim_{x \to 2^-} f(x)$ asks what f(x) appears to be doing as we approach x = 2 from the *left* (or negative) side (without looking at what it's actually doing at x = 2). Here, it looks like we're approaching 3. So, $\lim_{x \to 2^-} f(x) = 3$.

The one sided limits give us different answers. So, that means that $\lim_{x \to 2} f(x)$ does not exist. The only way for $\lim_{x \to 2} f(x)$ to exist would be if both $\lim_{x \to 2^+} f(x)$ and $\lim_{x \to 2^-} f(x)$ were equal. In which case, $\lim_{x \to 2} f(x)$ would equal that number.

Continuity

A function is continuous at x = 5 if:

$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^-} f(x) = f(5)$$

In plain english, a function is continuous if you can draw it without lifting your pen from the page. If the function has any jumps or gaps, it's discontinuous at that point.

Polynomials are continuous everywhere.

Rational functions are continuous everywhere on their domain. In other words, the only points of discontinuity will be when the denominator is equal to zero:

For example, the function $f(x) = \frac{1}{x^2 - 1}$ is discontinuous when $x^2 - 1 = 0$: when x = 1 or -1.

Differentiation

The derivative is defined in terms of limits. The derivative is a related function representing the rate of change of the original function.

The Four Step Process to find the derivative using limits is:

1. Find $f(a+h)$
2. Find $f(a+h) - f(a)$
3. Find $\frac{f(a+h) - f(a)}{h}$
4. Find $\lim_{h\to 0}\frac{f(a+h) - f(a)}{h}$

At each step, be sure to simplify the expression as much as possible before moving to the next step.

Constant Rule

If f(x) = constant, then f'(x) = 0

Power Rule

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$

When differentiating functions containing roots or terms of the form $\frac{1}{x^n}$, rewrite the terms using the rules:

$$\sqrt{x} = x^{1/2}$$
$$\sqrt[n]{x} = x^{1/n}$$

and

$$\frac{1}{x^n} = x^{-n}$$

Product Rule

The product rule is used to differentiate products of two functions:

$$\left(f \cdot g\right)' = f'g + g'f$$

Quotient Rule

The quotient rule is used to differentiate quotients of functions:

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

Chain Rule

The chain rule is used to differentiate composite functions

$$\left[f\left(g(x)\right)\right]' = f'\left(g(x)\right) \times g'(x)$$

Equation of a Tangent Line

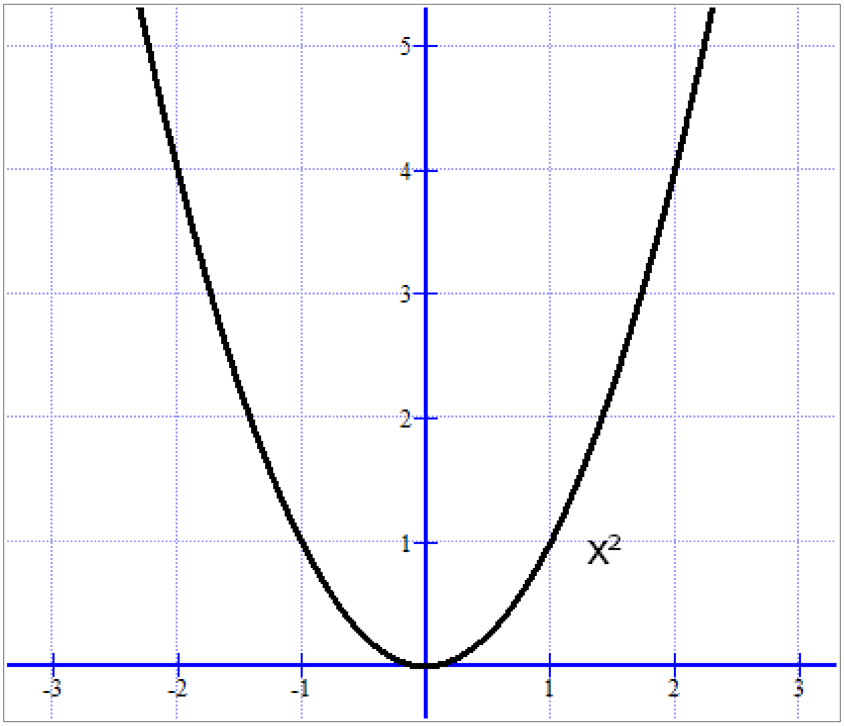
The general equation of a tangent line at a particular point is:

$y - y_o = m(x - x_o)$

m = slope of the tangent line
(x_o, y_o) = a point on the line

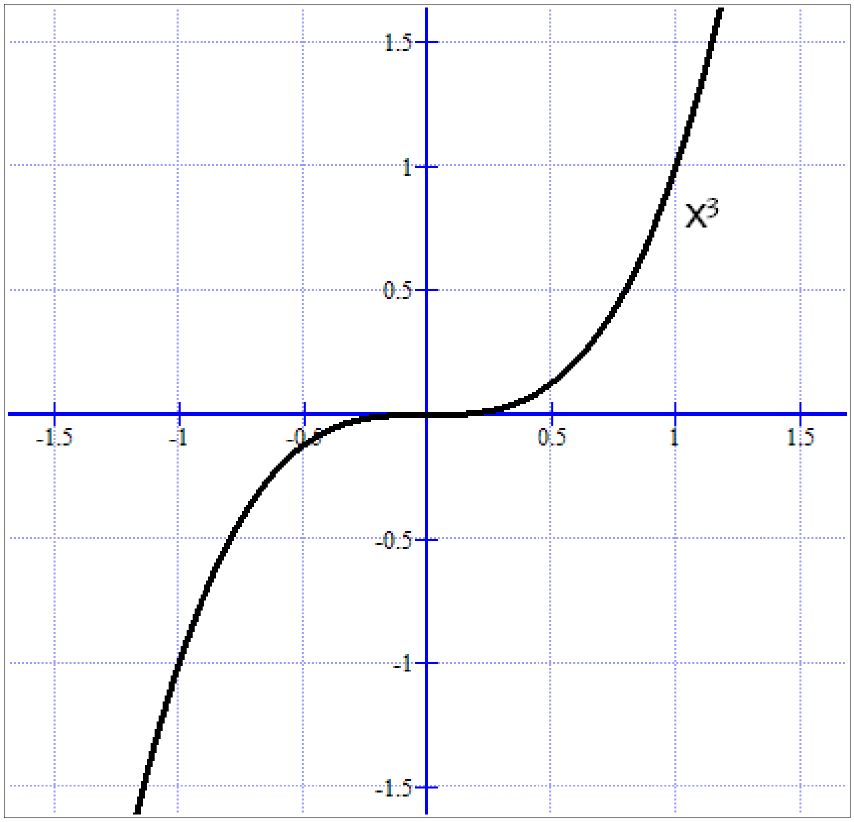
Good luck on your test!

The Parabola
$y = x^2$



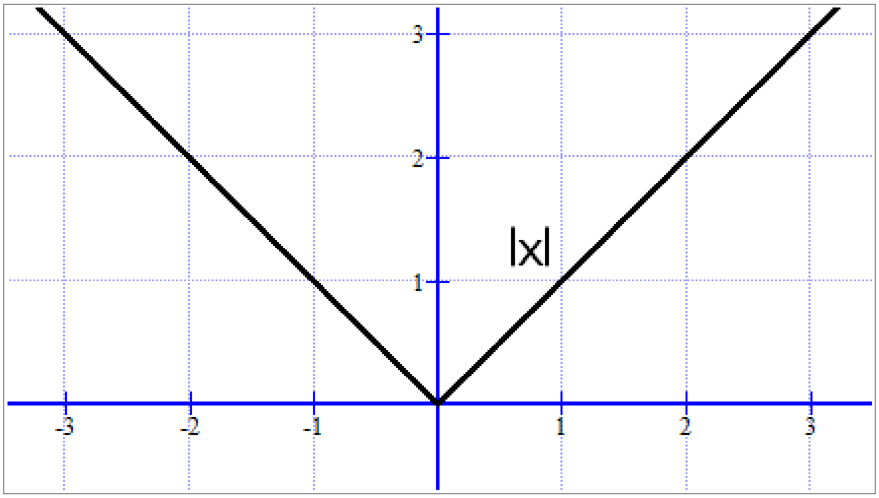
Function	$f(x) = x^2$
Domain	$(-\infty, \infty)$ (the entire real axis)
Range	$[0, \infty)$ (non-negative numbers)
x-intercept	(0, 0)
y-intercept	(0, 0)
Important Limits	$\lim_{x \to \infty} f(x) = \infty$ $\lim_{x \to -\infty} f(x) = \infty$

The Cubic Function
$y = x^3$



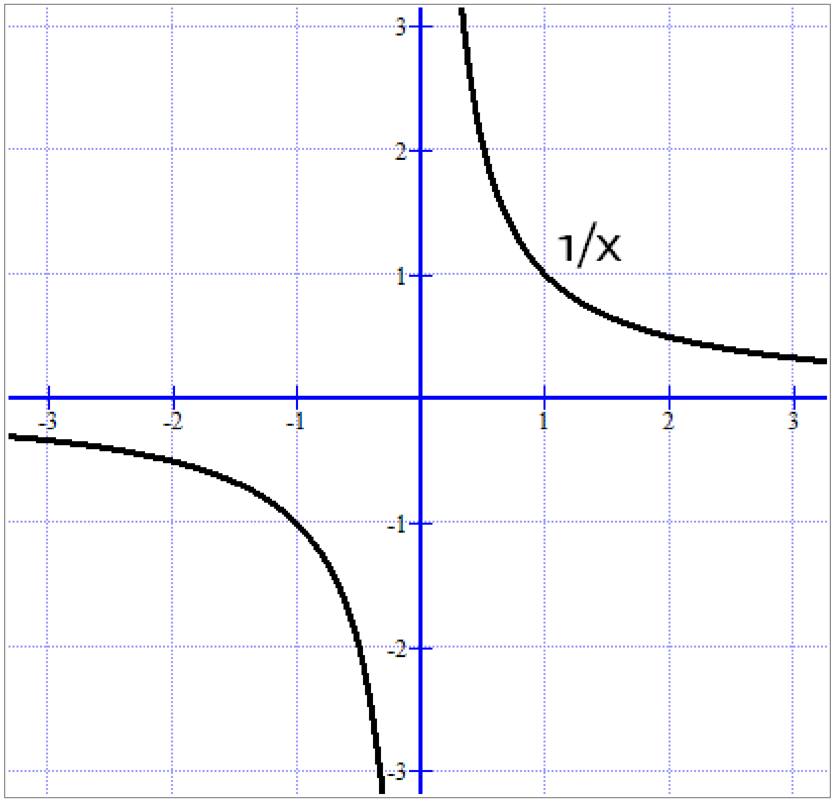
Function	$f(x) = x^3$
Domain	$(-\infty, \infty)$ (the entire real axis)
Range	$(-\infty, \infty)$
x-intercept	(0, 0)
y-intercept	(0, 0)
Important Limits	$\lim_{x\to\infty} f(x) = \infty$ $\lim_{x\to-\infty} f(x) = -\infty$

The Absolute Value Function
y = |x|



Function	$f(x) = \lvert x \rvert = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$
Domain	$(-\infty, \infty)$
Range	$[0, \infty)$
x-intercept	(0, 0)
y-intercept	(0, 0)
Important Limits	$\lim_{x \to \infty} f(x) = \infty$ $\lim_{x \to -\infty} f(x) = \infty$

The Reciprocal Function
y = 1/x



Function	$f(x)=\frac{1}{x}$
Domain	$(-\infty,0)\cup(0,\infty)$ (the entire real axis except for 0)
Range	$(-\infty,0)\cup(0,\infty)$
x-intercept	none
y-intercept	none
Important Limits	$\lim_{x\to\infty} f(x)=0$ $\lim_{x\to-\infty} f(x)=0$ $\lim_{x\to 0^+} f(x)=\infty$ $\lim_{x\to 0^-} f(x)=-\infty$