$$\frac{GM}{r^2} = \omega^2 r = 7.27\times10^{-5}$$

SECTION B

7. The time-independent Schrödinger equation for a single particle moving in a one-dimensional potential is given by

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi.$$

In addition to being a solution to this equation, what other conditions must the wavefunction $\psi(x)$ satisfy? [3]

For all parts of the remainder of this question, use the above Schrödinger equation specifically with $V(x) = -U\delta(x)$, where $\delta(x)$ is the Dirac delta function and U is a positive constant.

For this special potential, the derivative of ψ has a discontinuity at the origin resulting in the boundary condition

$$\frac{d\psi}{dx}\Big|_{x=0^+} - \frac{d\psi}{dx}\Big|_{x=0^-} = -\frac{2m}{\hbar^2}U\psi(0),$$

where $x = 0^-$ and $x = 0^+$ are positions immediately to the left and right of the origin.

(a) Find the general solutions to this Schrödinger equation for $E < 0$ in the two regions $x < 0$ and $x > 0$. [3]

By using these solutions with the usual boundary condition on ψ and the boundary condition on $d\psi/dx$ given above, find the energy eigenvalue and the normalized eigenfunction of the single bound state for $E < 0$. [5]

(b) Find the general solutions to this Schrödinger equation for $E > 0$ in the two regions $x < 0$ and $x > 0$. [3]

Using these solutions and the same boundary conditions as in (a) above, show that the proportion T of particles that are transmitted for a beam of particles incident from $x < 0$ on the above potential is [6]

$$T = \frac{2E\hbar^2}{2E\hbar^2 + mU^2}.$$

$E<0$

$U(r)T(r)$