5. The wavefunction of an electron in the hydrogen atom may be written in the form:

$$\psi = \frac{\chi(r)}{r} Y(\theta, \phi),$$

which leads to a Schrödinger-like equation for the function $\chi(r)$, with an *effective* potential given by

$$V_{\text{eff}}(r) = \frac{-e^2}{4\pi\epsilon_0 r} + \frac{l(l+1)\hbar^2}{2m_e r^2},$$

where m_e is the electron mass and l is the orbital quantum number. What is the physical significance of the two terms in this effective potential? [4]

Given that the function $Y(\theta, \phi)$ is chosen such that

$$\int_0^{2\pi} d\phi \int_{-\pi/2}^{\pi/2} d\theta \, |Y(\theta, \phi)|^2 \sin\theta = 1,$$

state the condition that χ must satisfy in order for the wavefunction ψ to be correctly normalized. [2]

6. Quantum mechanical operators are *Hermitian*. What mathematical property do the eigenvalues of a Hermitian operator possess? What do these eigenvalues correspond to? [2]

Explain what is meant if it is stated that the set of eigenfunctions $\{\phi_n\}$ of a Hermitian operator is *orthonormal*. [2]

In a one-dimensional system, an arbitrary function ψ may be expressed in terms of such a set of eigenfunctions, as

$$\psi(x) = \sum_n c_n \phi_n(x).$$

Give an expression for the expansion coefficient c_n. What is the interpretation of the value of $|c_n|^2$? [4]