11. Briefly describe the setup and results of the Stern-Gerlach experiment. [4]

The change in the Hamiltonian for orbital and spin interaction with an external magnetic field $\mathbf{B} = (0, 0, B_z)$ is

$$\Delta\hat{H} = \frac{\mu_B}{\hbar} B_z(\hat{L}_z + g\hat{S}_z).$$

Explain qualitatively the origins of the terms involving $\hat{L}$ and $\hat{S}$ and explain why the inclusion of the term involving $\hat{S}$ is necessary in order to describe the results of the Stern-Gerlach experiment. [4]

Suppose that ψ_{n,l,m,s,m_s} satisfies the time-independent Schrödinger equation for a hydrogen atom with energy E_0 in the absence of any magnetic field, where the quantum numbers $\{n, l, m, s, m_s\}$ have their usual meanings. What can you say about the result when the operators (i) $\hat{L}_z$ and (ii) $\hat{S}_z$ are applied to ψ_{n,l,m,s,m_s}? [2]

Show that in the presence of a magnetic field $(0, 0, B_z)$ it remains a solution but with energy

$$E_0 + \mu_B B_z(m + gm_s).$$

[5]

Hence write down the solution of the time-*dependent* Schrödinger equation corresponding to ψ_{n,l,m,s,m_s}, in the presence of the external field. [2]

Suppose the state of the atom at time $t = 0$ is a linear combination of solutions with two different values for m_s but the same values for all the other quantum numbers:

$$\Psi(t=0) = \frac{1}{\sqrt{2}}[\psi_{n,l,m,s,m_s=+1/2} + \psi_{n,l,m,s,m_s=-1/2}].$$

If the magnetic field is applied at time $t = 0$, show that the wave-function at subsequent times t is

$$\Psi(t) = \frac{\exp[-\mathrm{i}(E_0 + m\mu_B B_z)t/\hbar]}{\sqrt{2}}[\mathrm{e}^{-\mathrm{i}\mu_B B_z t/\hbar}\psi_{n,l,m,s,m_s=+1/2} + \mathrm{e}^{+\mathrm{i}\mu_B B_z t/\hbar}\psi_{n,l,m,s,m_s=-1/2}].$$

[3]

[You may assume that $g = 2$.]

END OF PAPER