Answer ALL SIX questions from Section A and THREE questions from Section B.

The numbers in square brackets at the right-hand margin indicate the provisional allocation of marks per sub-section of a question.

You may assume the following formulae throughout:

$$\text{Probability flux } \Gamma = -\frac{i\hbar}{2m}\left[\Psi^*\frac{\partial\Psi}{\partial x} - \Psi\frac{\partial\Psi^*}{\partial x}\right] = \frac{\hbar}{m}\text{Im}\left[\Psi^*\frac{\partial\Psi}{\partial x}\right].$$

SECTION A

[Part marks]

1. When X-rays are scattered from the free electrons in a material, there is a small shift in the wavelength of the X-ray beam. Is this shift to shorter or to longer wavelengths? [2]

   Name the *two* conservation laws needed to understand the wavelength shift. (You are *not* required to derive a formula for it.) Why are the results of the experiment important for our understanding of wave-particule duality? [4]

2. According to the Born interpretation, what is the significance of the quantity $|\Psi(x,t)|^2$ in quantum mechanics, where $\Psi(x,t)$ is the wavefunction of a one-dimensional system at position x and time t? Explain why Ψ should be normalized in order to make this interpretation consistent, and give (again in one dimension) the mathematical condition that has to be satisfied by a normalized Ψ. [4]

   Write down the time-dependent Schrödinger equation obeyed by Ψ when the particle moves under the influence of a potential $V(x,t)$. [3]

3. A particle of mass m in a one dimensional simple harmonic oscillator moves in a time-independent potential given by

$$V(x) = \frac{1}{2}kx^2.$$

   What is the angular frequency ω of the oscillation? State the energies of the lowest-energy stationary state (the ground state) and the first excited state. [3]

   How does your answer illustrate the concept of *zero-point energy*? [3]