8. A beam of particles having mass m moves freely in one dimension except that the potential energy is raised by a positive constant V_0 between the points $x = 0$ and $x = a$. In other words, the potential is

$$V(x) = \begin{cases} V_0 & (0 \leq x \leq a) \\ 0 & \text{(otherwise)} \end{cases}$$

Consider the case where the beam of particles approaches from the left, with an energy E exactly equal to V_0.

(a) Write down the time-independent Schrödinger equation for the regions $x < 0$ and $x > a$ where the potential is zero, and explain why suitable solutions are

$$\psi_1(x) = \mathrm{e}^{\mathrm{i}kx} + r\mathrm{e}^{-\mathrm{i}kx} \quad \text{for} \quad x < 0; \qquad \psi_3(x) = t\mathrm{e}^{\mathrm{i}kx} \quad \text{for} \quad x > a,$$

where t and r are constants. Give the value of k, and explain why is there no term proportional to $\mathrm{e}^{-\mathrm{i}kx}$ in the solution ψ_3. [5]

(b) Write down the time-independent Schrödinger equation for the region $0 \leq x \leq a$ and show that its general solution is of the form

$$\psi_2(x) = Ax + B,$$

where A and B are both constants. [3]

(c) Write down *four* matching conditions that the wavefunction must obey at the points $x = 0$ and $x = a$, explaining the origin of each. [6]

(d) Solve these matching conditions and show that

$$r = \frac{ka}{ka + 2\mathrm{i}}; \qquad t = \frac{2\mathrm{i}}{ka + 2\mathrm{i}}\mathrm{e}^{-\mathrm{i}ka}.$$

[4]

(e) Find the corresponding transmission probability. [2]