SECTION B

7. Starting from the time-dependent Schrödinger equation for the wavefunction $\Psi(x,t)$ of a particle of mass m moving in one dimension in a time-independent potential $V(x)$, show that separable solutions exist of the form

$$\Psi(x,t) = \psi(x)T(t)$$

and find the equations satisfied by the functions $\psi(x)$ and $T(t)$. Solve the equation for $T(t)$. [7]

Suppose the particle is free (i.e. acted upon by no forces) and hence take $V(x) = 0$. Show that a solution to the time-independent Schrödinger equation is

$$\psi_1(x) = Ae^{ikx} + Be^{-ikx},$$

where A and B are arbitrary constants. What is the corresponding energy? Explain why such a state is referred to as a *stationary state*, and show (using the expression for Γ given on the first page of the paper) that the corresponding probability flux is independent of time and equal to

$$\Gamma = \frac{\hbar k}{m}(|A|^2 - |B|^2).$$

[7]

Consider the same system, but prepared so that its state at time $t = 0$ is

$$\Psi_2(x, t=0) = Ae^{ikx} + Be^{-ik'x} \qquad \text{where} \qquad k \neq \pm k'.$$

Write down the subsequent evolution of $\Psi_2(x,t)$ under the time-dependent Schrödinger equation, in terms of the quantities $E = \hbar^2k^2/2m$ and $E' = \hbar^2k'^2/2m$. Compute the corresponding probability density for the system. Is this a stationary state? Give reasons for your answer. [6]

$E\Psi = i\hbar\frac{\partial\Psi}{\partial t}$