Answer ALL SIX questions from Section A and THREE questions from Section B.

The numbers in square brackets at the right-hand margin indicate the provisional allocation of marks per sub-section of a question.

You may assume the following values:

Planck constant $h = 6.63 \times 10^{-34}$ J s; $\hbar = 1.05 \times 10^{-34}$ J s;
Electronic charge $e = 1.60 \times 10^{-19}$ C;
Mass of electron $m_e = 9.11 \times 10^{-31}$ kg;

SECTION A

[Part marks]

1. The time-dependent Schrödinger equation for a particle moving in one dimension can be written
$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi}{\partial t}.$$
Define the quantities V and Ψ appearing in this equation. [3]

Give an expression involving *one* of the above quantities for the probability that the particle may be found in a small region of space between the positions x and $x + \delta x$. [3]

2. The function
$$\psi(x) = Ce^{ikx},$$
where C and k are constants, is a solution to the time-independent Schrödinger equation for a particle moving in free space in one dimension.

How is the quantity k related to the de Broglie wavelength of the particle? [2]

The function $\psi(x)$ is an eigenfunction of the momentum operator
$$\hat{p} = -i\hbar\frac{\partial}{\partial x}.$$
By substituting $\psi(x)$ into the defining equation for an eigenfunction, find the corresponding eigenvalue. [4]

3. Let the functions $\{\phi_n\}$ be the eigenfunctions of some Hermitian operator. Explain what is meant if it is stated that this set of functions is *orthonormal*. [4]

The expansion postulate allows us to express an arbitrary function ψ in terms of such a set of eigenfunctions. What is the general form of such an expression? (You are *not* required to evaluate the quantities appearing in it.) [3]