**UNIVERSITY COLLEGE LONDON**

**EXAMINATION FOR INTERNAL STUDENTS**

| | | |
|---|---|---|
| MODULE CODE | : | **PHAS1224** |
| ASSESSMENT PATTERN | : | **PHAS1224A** |
| MODULE NAME | : | **Waves, Optics and Acoustics** |
| DATE | : | **12-May-08** |
| TIME | : | **10:00** |
| TIME ALLOWED | : | **2 Hours 30 Minutes** |

2007/08-PHAS1224A-001-EXAM-162



**TURN OVER**

**Answer ALL SIX questions from section A and THREE questions from section B.**

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

---

You may need these:

*Law of cosines*:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha,$$

where $\alpha$ is the angle across from the side of length $a$.

*Law of sines*:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma},$$

where the sides of length $a$, $b$ and $c$ are across from the angles $\alpha$, $\beta$ and $\gamma$, respectively.

*Addition of cosines*:

$$\cos A + \cos B = 2\cos\left[\frac{1}{2}(A+B)\right]\cos\left[\frac{1}{2}(A-B)\right].$$

*Addition of sines*:

$$\sin A + \sin B = 2\sin\left[\frac{1}{2}(A+B)\right]\cos\left[\frac{1}{2}(A-B)\right].$$

*Binomial expansion of exponential*

$$e^{\pm\alpha x} \approx 1 \pm \alpha x \mp \ldots.$$

---

## SECTION A

**[Part marks]**

1. (a) Describe three ways in which light can be linearly polarized. **[3]**

   (b) i. State the Law of Malus. **[2]**

   ii. Unpolarized light of intensity $I_0$ is incident on a linear polarizing sheet. What is the intensity of the outgoing light with respect to $I_0$? **[1]**

2. (a) Write down the partial differential equation describing wave motion in one dimension. Define your variables. **[4]**

   (b) A uniform string under tension $T$ has a mass $m$, diameter $d$ and length $L$. At what speed would a wave disturbance travel down the string? **[3]**

3. (a) Write down equations which express the Law of Reflection and the Law of Refraction for a ray of light striking a flat interface. Describe the symbols used and outline any condition(s) which must apply. **[3]**

   (b) Light, travelling in a medium with index of refraction $n_1$, is incident upon a medium with index of refraction $n_2$.

   i. Under what condition for the indices does *total* internal reflection occur? **[2]**

   ii. Write down the formula for the critical angle, $\theta_c$, at which *total* internal reflection occurs. **[2]**

4. (a) Two audio oscillations have the same amplitude and frequencies, $f_1$ and $f_2$, which are very close in value. If the oscillations are added,

   i. what is the frequency of the resulting oscillations? [1]

   ii. what is the *beat* frequency? [1]

   (b) Two phasors are represented by $3e^{i\omega t}$ and $4e^{i(\omega t+\frac{\pi}{2})}$.

   i. Draw a diagram of the two phasors and their sum, $Ae^{i(\omega t+\phi)}$. [2]

   ii. Determine the values of $A$ and $\phi$. [3]

5. (a) Describe Rayleigh's criterion for the resolution of images formed by a slit. [3]

   (b) State the formula that gives the minimum angular separation between two images which can just be resolved, at wavelength $\lambda$, by a slit of width $w$. [2]

   (c) State how this expression from Question 5b is modified for a circular aperture of diameter $D$. [2]

6. (a) Which common condition of the eye is illustrated by Figure 1? [2]

   (b) What type of lens should be used to correct this condition? [2]

   (c) What is meant by the *near point* of the eye? [2]

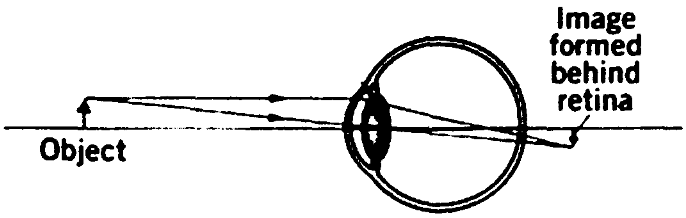


**Figure 1** Question 6.

SECTION B

7. The Fraunhofer diffraction pattern for an array of $N$ parallel slits is shown in Figure 2. The slits have width $b$ and their centres are a distance $a$ apart. The intensity is given by

$$I(\theta) = \frac{I(0)}{N^2}[\frac{\sin\beta}{\beta}]^2[\frac{\sin N\alpha}{\sin\alpha}]^2$$

where, with $k = 2\pi/\lambda$,

$$\beta \equiv \frac{kb}{2}\sin\theta \qquad\qquad \alpha \equiv \frac{ka}{2}\sin\theta$$

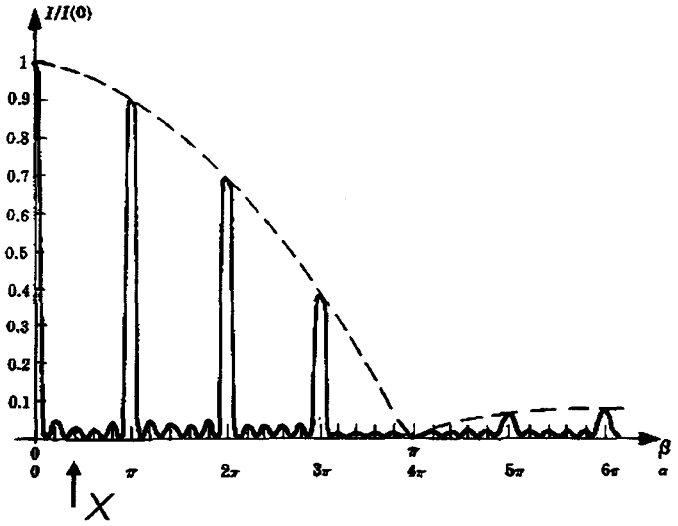


**Figure 2** Question 7.

(a) How many slits are in the array? Give your reasons. **[3]**

(b) What is the ratio $a/b$? Give your reasons. **[4]**

(c) Show that the second subsidiary peak adjacent to the first principal maximum, marked at $X$ by the arrow in Figure 2, corresponds approximately to $\alpha = \frac{5\pi}{2N}$. **[4]**

(d) Compute the intensity of the second subsidiary peak adjacent to the first principal maximum, marked at $X$ by the arrow in Figure 2, in terms of $I(0)$. **[4]**

(e) Show that the angular width, $\Delta\theta$, of the central principal maximum of a diffraction grating pattern is given by: **[5]**

$$\Delta\theta = \frac{2\lambda}{Na},$$

where $Na$ is the width of the grating.

8. (a) i. Describe what is meant by the phase velocity, $v_p$, and group velocity, $v_g$, of waves. [2]

ii. In what type of environment are the two velocities equal? What is an example of such an environment? [2]

iii. Write expressions for $v_p$ and $v_g$ in terms of $\omega$ and $k$ and show that $v_g$ can be written as [3]

$$v_g = v_p + k\frac{\mathrm{d}v_p}{\mathrm{d}k}.$$

(b) The dispersion relation for waves in water of depth $x$, which involves the acceleration due to gravity $g$, the surface tension $\Gamma$, and the water density $\rho$, is

$$\omega^2 = \left(\frac{g + \Gamma k^2}{\rho}\right) k \left(\frac{e^{kx} - e^{-kx}}{e^{kx} + e^{-kx}}\right).$$

Show that the phase velocity

i. in the limits of shallow water, is given by [5]

$$v_p = \sqrt{x\left(\frac{g + \Gamma k^2}{\rho}\right)};$$

and,

ii. in the limits of very deep water, is given by [4]

$$v_p = \sqrt{\frac{1}{k}\left(\frac{g + \Gamma k^2}{\rho}\right)},$$

(c) An ionized gas or plasma is a dispersive medium for electromagnetic waves. Given that the dispersion relation is

$$\omega^2 = \omega_p^2 + c^2k^2$$

where $\omega_p$ is the constant plasma frequency, show that [4]

$$c^2 = v_p v_g.$$

9. A camera lens is usually a combination of two or more single lenses. Consider a camera lens consisting of a diverging lens, with $|f_1| = 120$ mm, and a converging lens, with $|f_2| = 42$ mm, spaced 60 mm apart. A 10-cm-tall object is 500 mm from the first lens.

(a) What are the location, magnification and orientation of the image formed by the first, diverging, lens? **[6]**

(b) What are the location, magnification and orientation of the final image formed by the lens combination? **[5]**

(c) Draw a pictorial representation of the images formed by the combination camera lens. Show clearly the rays used to locate the images. **[5]**

(d) What is the focal length of a *single* lens that could produce an image in the same location if placed at the midpoint of the lens combination? **[4]**

10. (a) The Doppler effect is demonstrated by tying a 600 Hz sound generator to a 1.0-m-long rope and whirling it around in a horizontal circle at 100 rpm. What is the difference between the highest and lowest frequencies heard by a student in the classroom? Assume that the speed of sound in air is 343 m s$^{-1}$. **[6]**

(b) i. If you are given two sound intensities, $I_1$ and $I_2$, how is the difference in the sound levels $\beta_1$ and $\beta_2$ related to the intensities? **[3]**

ii. By what factor has the intensity increased if the sound level has risen by 10 dB? **[2]**

(c) Two loudspeakers, at the same height, are 2.0 m apart and in phase with each other. Both emit 705 Hz sound waves into a room where the speed of sound is 343 m s$^{-1}$. A listener stands 5.0 m in front of the loudspeakers and 2.0 m to one side of the centre.

i. Is the interference at this point constructive, destructive, or something in between? **[6]**

ii. How would the situation differ if the loudspeakers are $\pi$ out of phase? **[3]**

11. In this question, assume that the speed of light is given by $c = 3 \times 10^8$ m s$^{-1}$ and that the speed of sound in air is 343 m s$^{-1}$.

(a) Microwaves form standing waves in a microwave oven. Cold spots in a microwave oven are found to be 6.0 cm apart. What is the frequency of the microwaves? **[3]**

(b) i. A shower stall, e.g., Figure 3, closed at the top, is 2.45 m tall. For what frequencies less than 300 Hz are there standing sound waves in the shower stall (the door is closed)? **[4]**

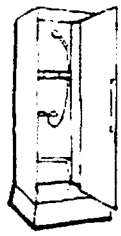

**Figure 3**
Question 11b.

ii. If the top of the stall is open, for what frequencies less than 300 Hz are there standing sound waves in the shower stall? **[4]**

iii. Draw the graphical representations of the standing waves for the first two frequencies found in Questions 11(b)i and 11(b)ii. **[4]**

(c) i. What are the three longest wavelengths for standing waves on a 240-cm-long string that is fixed at both ends? **[3]**

ii. If the frequency of the second-longest wavelength is 50 Hz, what is the frequency of the third-longest wavelength? **[2]**



**END OF PAPER**

## UNIVERSITY COLLEGE LONDON

## EXAMINATION FOR INTERNAL STUDENTS

| | | |
|---|---|---|
| MODULE CODE | : | **PHAS1224** |
| ASSESSMENT PATTERN | : | **PHAS1224A** |
| MODULE NAME | : | **Waves, Optics and Acoustics** |
| DATE | : | **19-May-09** |
| TIME | : | **10:00** |
| TIME ALLOWED | : | **2 Hours 30 Minutes** |

2008/09-PHAS1224A-001-EXAM-97

**Answer ALL SIX questions from section A**
**and THREE questions from section B.**

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

---

You *may* need these:

*Double-angle formulae*:

$$\sin 2\theta = 2\cos\theta \sin\theta,$$

$$\cos 2\theta = 2\cos^2\theta - 1.$$

*Law of cosines*:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha,$$

where $\alpha$ is the angle across from the side of length $a$.

*Law of sines*:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma},$$

where the sides of length $a$, $b$ and $c$ are across from the angles $\alpha$, $\beta$ and $\gamma$, respectively.

*Angle sum and difference identities*:

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B,$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B.$$

*Threshold intensity*:

$$I_0 = 10^{-12}\ \mathrm{W\ m^{-2}}$$

---

## SECTION A

**[Part marks]**

1. (a) Write down expressions for the phase velocity $v_p$ and group velocity $v_g$ of a wave in terms of the angular frequency $\omega$ and the wavevector $k$. **[2]**

   (b) A number of waves combine to give a modulated wave. Sketch this modulated wave and indicate on your sketch which part moves with $v_p$ and which part moves with $v_g$. **[2]**

   (c) At what speed does energy propagate in the wave? **[1]**

2. (a) The six strings of a guitar are of the same length and are under nearly the same tension, but have different thicknesses. On which string do waves travel the fastest? Briefly explain your answer. **[3]**

   (b) While a guitar string is vibrating, you gently touch the midpoint of the string to ensure that the string does not vibrate at that point. Briefly explain which normal modes *cannot* be present on the string while you are touching it in this way? **[3]**

3. (a) Draw a diagram of a Michelson interferometer, and label its main features. **[4]**

   (b) Under what conditions will circular fringes be observed with the apparatus you have drawn? **[2]**

   (c) Explain why a compensator plate is necessary if a Michelson interferometer is to be used with non-monochromatic light. **[2]**

4. (a) In Figure 1, light rays from an object strike an optical element (obscured by the dark rectangle) and form an inverted image. What type of optical element is it? Why? **[3]**

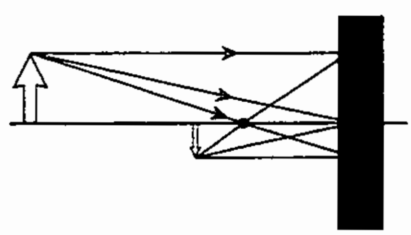

**Figure 1** Question 4a

   (b) A biconcave thin lens of focal length magnitude $f$ forms an image of the moon. Where is the image located? **[3]**

5. (a) State the changes of phase which occur when light is reflected [2]

i. in air from an air-glass interface; and,
ii. in glass from a glass-air interface.

(b) i. Define the index of refraction of a medium, $n$, as the ratio of two velocities. [2]
ii. The index of refraction can be complex. What is the significance of this? [2]

(c) Light strikes an interface between the incident medium, with an index of refraction $n_i$, and air, with an index of refraction $n_t = 1.00$, such that $n_i > n_t$. The reflected intensities of the components of the light both parallel (|| – solid line) and perpendicular ($\perp$ – dashed line) to the interface, as a function of the incident angle ($\theta_i$), are shown in Figure 2.

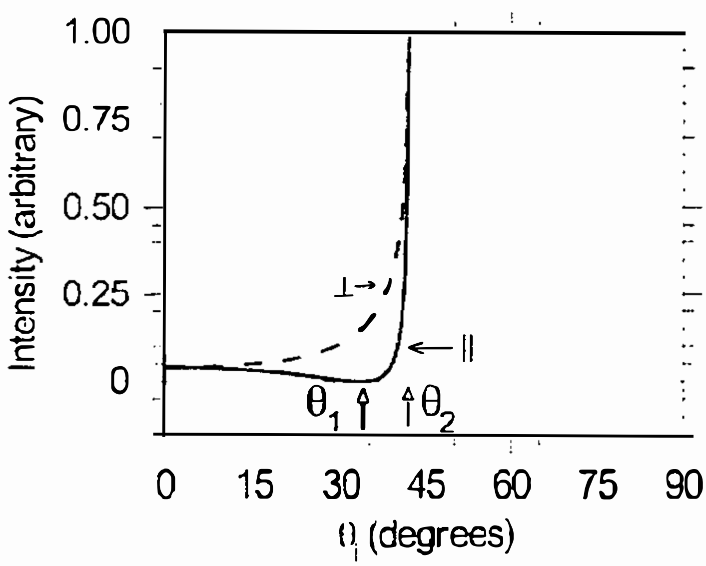


**Figure 2** Question 5c.

Two characteristic angles are also marked, $\theta_1 = 33.7°$ and $\theta_2 = 41.8°$.

What is the significance of the angles $\theta_1$ and $\theta_2$, marked in the diagram by arrows? [3]

6. State

(a) Fermat's principle; and, [3]

(b) Huygen's principle. [3]

## SECTION B

7. (a) i. Describe four ways in which light can be linearly polarized. **[4]**

ii. Naturally unpolarized light, with intensity $I_0$, is incident on a set of three polarizing sheets. The three sheets are arranged in-line perpendicular to the direction of the light. The polarizing direction of the first sheet is parallel to the horizontal. The polarizing direction of the third sheet is parallel to the vertical. At what angle from the horizontal must the polarizing direction of the middle sheet be set so that the final outgoing intensity is given by **[6]**

$$I = \frac{3}{32} I_0 \text{ ?}$$

(b) Two lightbulbs, which emit light with $\lambda = 600$ nm, are 1.0 m apart. From what distance can these lightbulbs be marginally resolved by a small telescope with a 4.0 cm diameter objective lens? **[5]**

(c) A light ray travels from medium 1, with index of refraction $n_1$, to medium 3, with index of refraction $n_3$, through medium 2, with index of refraction $n_2$, as shown in Figure 3. For these media, compare qualitatively the magnitudes of $n_1$, $n_2$ and $n_3$. **[5]**

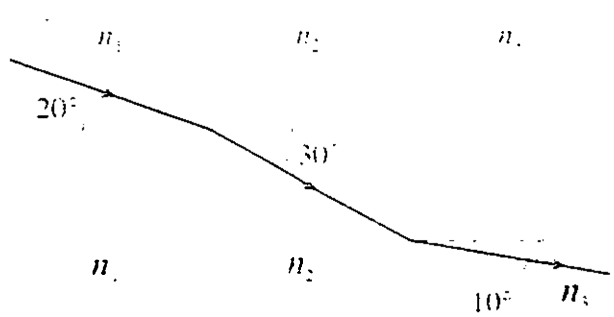


**Figure 3** Question 7c.

8. (a) A diffraction grating with 500 slits per millimetre is illuminated with light from a gas discharge lamp which contains more than one wavelength. Diffraction maxima, which are at least first-order, are observed at 14.73°, 16.77°, 30.57°, 35.24°, 49.72° and 59.94°.

i. What is the spacing between adjacent slits? [2]

ii. What wavelengths of light are present in the lamp's spectrum? [3]

(b) Figure 4 shows the viewing screen in a double-slit experiment. Fringe C is the centre fringe.

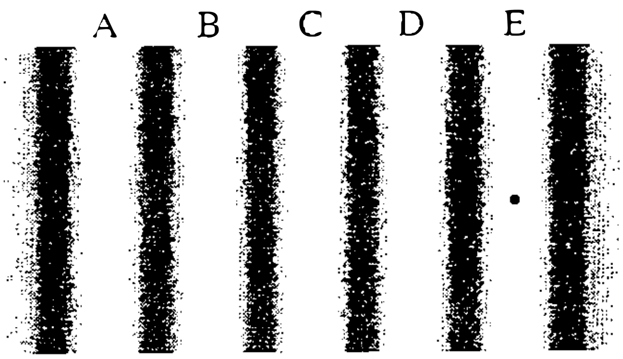


**Figure 4** Question 8b.

i. What would happen to the fringe spacing if the wavelength of the light is decreased? Why? [2]

ii. What would happen to the fringe spacing if the spacing between the slits is decreased? Why? [2]

iii. What would happen to the fringe spacing if the distance to the screen is decreased? Why? [2]

iv. Suppose the wavelength of the light is 500 nm. How much farther is it from the dot on the screen in the centre of fringe E to the left slit than it is from the dot to the right slit? [3]

(c) A thin wedge-shaped film of methyl alcohol ($n_f = 1.3290$) is formed between two flat plates of glass. Yellow sodium light of vacuum wavelength 589 nm falls nearly normally on the film, generating fringes separated by 0.2 mm. Determine the wedge angle in degrees. [6]

9. (a) You need to improvise a magnifying glass to read some very tiny print. Should you borrow the eyeglasses from your hyperopic friend or from your myopic friend? Why? [4]

(b) The image of a slide on a screen is blurry because the screen is in front of the image plane. To focus the image, should you move the lens toward the slide or away from the slide? Why? [4]

(c) A 2.0-cm-tall object is 20 cm to the left of a converging lens with a focal length of magnitude 10 cm. A diverging lens with a focal length of magnitude 5 cm is 30 cm to the right of the first lens.

i. Calculate the position of the final image. [6]

ii. Calculate the overall lateral magnification $M$ of the two-lens combination and the height of the final image. [4]

iii. Is the final image upright or inverted? [2]

10. (a) A transverse wave with a wavelength of 1.2 m is travelling along a string with a speed of 240 $\text{ms}^{-1}$.

i. Calculate the angular frequency and angular wavenumber of the wave. [4]

ii. If the amplitude of the wave is 1.0 mm, calculate the value of the displacement, $y(x,t)$, at $x = 0.3$ m and $t = 0.0025$ s. [4]

(b) A second transverse wave with the same amplitude, wavelength and frequency also travels along the same string. The phase difference between the two waves is $\pi/3$. Using phasors, calculate:

i. the amplitude of the total displacement due to both waves; and, [3]

ii. the value of the total displacement due to both waves at $x = 0.0$ m and $t = 0.0$ s. [3]

(c) The fundamental frequency of an open organ pipe corresponds to middle C, or 261.6 Hz. The third resonance of a closed pipe has the same frequency. What are the lengths of the two pipes? The speed of sound in air is 343 m $\text{s}^{-1}$. [6]

11. (a) A blender making a smoothie produces a sound intensity level of $\beta = 83$ dB.

i. What is the intensity of the sound, $I$? [4]

ii. What would the sound intensity level be if two more blenders were turned on? [4]

(b) i. A source emits a sound wave with frequency $f_S$. The frequency with which a listener hears the sound wave is $f_L$. The speed of sound in the medium is $v$. If the listener is moving toward a stationary source with velocity $v_L$, the observed frequency is

$$f_L = \left(1 + \frac{v_L}{v}\right) f_S.$$

If a source moves toward a stationary listener with velocity $v_S$, the observed frequency is

$$f_L = \left(\frac{1}{1 - \frac{v_S}{v}}\right) f_S.$$

Derive **one** of the above equations. [6]

ii. A whistle you use to call your dog has a frequency of 21 kHz, but your dog is ignoring it. You suspect the whistle may not be working, but you can't hear sounds above 20 kHz. To test it, you ask a friend to blow the whistle, then you hop on your bicycle. The speed of sound in air is 343 m $s^{-1}$.

A. To determine if the whistle is working, in which direction should you ride: toward or away from your friend? Why? [2]

B. At what minimum speed should you ride to determine if the whistle is working? [4]

 **END OF PAPER**

# UNIVERSITY COLLEGE LONDON

# EXAMINATION FOR INTERNAL STUDENTS

| | | |
|---|---|---|
| MODULE CODE | : | **PHAS1224** |
| ASSESSMENT PATTERN | : | **PHAS1224A** |
| MODULE NAME | : | **Waves, Optics and Acoustics** |
| DATE | : | **11-May-10** |
| TIME | : | **10:00** |
| TIME ALLOWED | : | **2 Hours 30 Minutes** |

**Answer ALL SIX questions in Section A and THREE questions from Section B**

The numbers in square brackets at the right-hand side of the text indicate the provisional allocation of maximum marks per question or sub-section of a question.

You may find the following constant useful.
Speed of sound in air is 330 m/s.

# SECTION A

1. (a) Two phasors are defined by $5e^{\imath\omega t}$ and $4e^{\imath\omega t+\pi/4}$. Draw a diagram of the two phasors and their sum, $Ce^{\imath\omega t+\phi}$ and calculate $C$ and $\phi$. [4]

   (b) If two driving forces of the same amplitude and phase but slightly different frequencies $\omega_1$ and $\omega_2$ are applied to a simple harmonic oscillator, what is the name of the resulting behaviour ? Describe it *briefly* in terms of $\omega_1$ and $\omega_2$. [3]

2. (a) Write down the differential equation describing wave propagation in one dimension, defining your variables. [4]

   (b) In a pipe of diameter $d$ and length $L$ containing gas of density $\rho$ and compressibility $\kappa$ at pressure $P$, at what speed would sound waves travel ? [2]

3. (a) State Fermat's principle for the propagation of light. [2]

   (b) Consider two media of refractive index $n_i$ and $n_t$ separated by a flat plane. A light ray in the first medium is incident on the boundary at an angle $\theta_i$ to the normal to the boundary, and continues in the second medium making an angle $\theta_t$ to the normal. Using Snell's law, $n_i \sin\theta_i = n_t \sin\theta_t$, explain why there may sometimes be no light transmitted in the case $n_t < n_i$ and calculate the incident angle at which this happens if medium $i$ is glass, with $n_i = 1.5$, and medium $t$ is water, with $n_t = 1.33$. [4]

4. (a) State the Law of Malus for polarised light. [2]

   (b) Unpolarised light of intensity $I_0$ is incident on a linear polarising sheet with its transmission in the vertical direction. What is the intensity of the transmitted light, $I_1$? [2]

   (c) The light coming from the polariser described in part (b) goes through a second polariser with orientation rotated by $\pi/4$ in the clockwise direction and a third with orientation rotated by a further $\pi/4$ in the clockwise direction. Explain qualitatively the change in the intensity of transmitted light if the second and third polarisers swap places? [4]

5. (a) Briefly describe Rayleigh's criterion for the resolution of images formed by a thin slit, give the minimum angular separation that can be resolved at wavelength $\lambda$ if the slit has width $a$, and state how the answer is modified if the aperture is instead a circle with diameter $D$. [4]

   (b) Your eye has a pupil diameter of roughly 2mm and the lens is about 20mm from the retina. The separation of the receptors in the retina is about $10^{-6}$m. Explain why there is nothing to be gained from having the receptors much closer together. [3]

6. (a) State the changes in phase which occur when light is reflected at a boundary

   i. when travelling from a region of low refractive index $n_1$ towards one of high refractive index $n_2$ i.e. when $n_2 > n_1$, and

   ii. when travelling from a region of high refractive index $n_1$ towards one of low refractive index $n_2$ i.e. when $n_2 < n_1$. [2]

   (b) A glass surface with refractive index $n_g = 1.5$ is coated with a material with refractive index $n_{ar} = 1.25$ which acts as an anti-reflection coating. Explain how this helps reduce reflection and find the required thickness of the coating to do this most effectively for incident light of wavelength $\lambda = 500$nm. [4]

# SECTION B

7. (a) When an ambulance approaches an observer, the pitch of the siren is raised as is the frequency of alternation between high and low pitch. Show that the frequency for the observer is given by:

$$f_L = \left(\frac{v}{v - v_S}\right) f_S$$

where the ambulance is moving with speed $v_S$, the speed of sound in air is $v$ and the frequency of the source is $f_S$. [6]

(b) The ambulance is travelling at 96 kilometres per hour, the pitch of the siren is 440Hz and the frequency of alternation is 60Hz. What pitch and frequency of alternation does the observer find ? [4]

(c) When the ambulance is 100 metres away from the observer, the sound level is 90 dB. Given that sound *intensity* changes with the inverse of the *square* of the distance, what is the sound level when the ambulance is 50 metres away from the observer ? [6]

(d) What will happen to the total sound level at the observer if another ambulance (with the same siren intensity) is 100 metres away when the first is 50 metres away ? [4]

8. (a) The equation of motion for the displacement $x$ of a driven oscillator can be written:

$$m\frac{d^2x}{dt^2} + kx = F_0 e^{i\omega t}$$

where $m$ is the mass, $k$ is the spring constant and $F_0$ and $\omega$ are the amplitude and frequency of the driving force.

i. What is the natural frequency of this oscillator ? [2]

ii. Show that the motion $x = Ae^{iBt}$ is a solution of the motion, and find values for $A$ and $B$. [4]

iii. For what value of $B$ will the value of $A$ become undefined, and physically what does this represent ? [2]

iv. What physical effect would a term $b\frac{dx}{dt}$ on the left hand side represent ? Explain whether there will be values of B where A becomes undefined if such a term is introduced and why. [2]

(b) Two undriven oscillators with displacements $x_1$ and $x_2$ have the same mass $m$ and spring constant $k$ and no dissipation terms. They are connected by a spring with constant $K$.

i. Write down the coupled equations of motion for $x_1$ and $x_2$ in terms of $k$ and $m$. [4]

ii. Show that the equations of motion can be transformed to take the form:

$$\begin{aligned} m\frac{d^2q_a}{dt^2} &= -kq_a \\ m\frac{d^2q_b}{dt^2} &= -(k+2K)q_b \end{aligned}$$

and define $q_a$ and $q_b$. [6]

9. (a) What are the characteristic impedance, $Z_0$, and the speed, $c$, for waves propagating on a string under tension $T$ with mass per unit length $\mu$ ? [4]

(b) The reflection coefficient for waves propagating from a medium with impedance $Z_1$ to a medium with impedance $Z_2$ (e.g. from one string to another) is:

$$R = \frac{(Z_1 - Z_2)}{(Z_1 + Z_2)}$$

Explain, with reference to the physics of each situation, what happens when:

i. $Z_1 \gg Z_2$ [2]

ii. $Z_1 \ll Z_2$ [2]

iii. $Z_1 = Z_2$ [2]

(c) In my building, we have a handrail which consists of a piece of steel one metre long, horizontally supported by two vertical posts at either end. A standing wave can be set up in the handrail by tapping the steel bar.

i. Give a general equation for the modes which can be supported, in terms of the length, $L$. Define any other symbols which you may require. [3]

ii. What are the wavelengths of the first three modes in the steel bar ? [3]

iii. If a third vertical post is added so that it attaches to the centre of the steel bar, what are the wavelengths of the first three modes now ? Why ? [4]

10. (a) State Huygens' principle. [2]

(b) Explain the conditions under which Fraunhofer diffraction takes place. [2]

(c) Using Huygens' principle and assuming that the conditions stated in (b) are satisfied one can determine diffraction patterns. Show the pattern due to two very thin slits with separation $d$, formed on a screen at a distance very much further than $d$ from the slits, and at angle $\theta$ to the normal to the plane of the slits, has intensity $4I_0 \cos^2(\delta/2)$. $I_0$ is the intensity arising from one slit and $\delta = (2\pi/\lambda)d\sin\theta$. [4]

(d) Extending to the case of a diffraction grating with $N$ slits the intensity becomes

$$I = I_0 \frac{\sin^2(N\beta)}{\sin^2(\beta)},$$

where $\beta = \delta/2$. Two lines of wavelength $\lambda$ and $\lambda + \Delta\lambda$ are said to just be resolved by the grating if the principal maximum of one lies on the first minimum of the other. Show that for the first order of lines this results in a chromatic resolving power of $\lambda/\Delta\lambda = N$. [4]

(e) The transmission function for a Fabry-Perot etalon is given by

$$T_e = \frac{1}{1 + F\sin^2(\gamma)}.$$

$\gamma$ is $(2\pi/\lambda)t\cos\theta$, where $t$ is the distance between the parallel mirrors in the etalon. The coefficient of finesse $F$ is equal to $4R/(1-R)^2$, where $R$ is the reflectance of the mirrors. In this case two lines of different wavelength $\lambda$ and $\lambda + \Delta\lambda$ are said to be resolved if the maximum of one corresponds to the point at which the other has fallen to half maximum. Show that this occurs if $|\Delta\gamma| \approx 1/\sqrt{F}$ where the equality becomes better as $R$ approaches 1. [5]

(f) Following from this it can be seen that for the 1st order maximum we have the condition $\lambda/\Delta\lambda = \pi\sqrt{F}$. Using this result comment on how good the mirror in an etalon must be to compete with a grating of 1000 lines, i.e. find the required value of $R$. [3]

11. (a) Consider a spherical concave mirror of radius of curvature $R$ with an object of height $h$ with its base on the principal axis and to the left of the centre of curvature.

i. Show the inverted image formed by the mirror by drawing the two rays from the top of the object which either go through the centre of curvature or hit the mirror at the intersection with the principal axis. [2]

ii. Consider the four right-angled triangles which have either the object or image as the opposite side and in each case either the centre of curvature or the intersection of the principal axis and the mirror as an apex. Show that the object distance $p$ and image distance $q$ are related by [3]

$$\frac{p-R}{p}=\frac{R-q}{q}.$$

Taking the focal length $f$ of the mirror to be $f = R/2$, derive the mirror equation. [2]

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}.$$

(b) A compound microscope is made from two converging lenses, an objective with focal length $f_o = 5\text{mm}$ and an eyepiece with focal length $f_e = 30\text{mm}$.

i. Draw a diagram (not to scale) of the microscope. Show the lenses and the approximate position of the real image $I_1$ formed by an object when placed just outside the focal point of the objective and of the final image $I_2$ formed by the eyepiece. In particular, indicate the required position of $I_1$ relative to the eyepiece such that the magnified image $I_2$ seen by the observer will be virtual and inverted. The diagram should show important rays, but need not be to scale or highly detailed. [4]

ii. An object is placed 5.1mm from the objective. Find the position of the image formed by the objective by using the lens equation. [2]

iii. We want the image of the object from the second lens to be at 250mm from the eyepiece. Calculate how far the image formed by the objective must be from the eyepiece in order to achieve this. [2]

iv. Use the results obtained in the last two parts of the question to find the required separation of the lenses $L$ and also find the exact lateral magnification $M$ of the microscope. [3]

v. How does your answer for the magnification compare to the approximate expression from the lectures of $M = -(250L)/(f_o f_e)$? [2]

# UNIVERSITY COLLEGE LONDON

# EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : **PHAS1224**

ASSESSMENT PATTERN : **PHAS1224A**

MODULE NAME : **Waves, Optics and Acoustics**

DATE : **27-May-11**

TIME : **10:00**

TIME ALLOWED : **2 Hours 30 Minutes**

**Answer ALL SIX questions in Section A and THREE questions from Section B**

The numbers in square brackets at the right-hand side of the text indicate the provisional allocation of maximum marks per question or sub-section of a question.

You may find the following formula useful.

$$\cos\alpha + \cos\beta = 2\cos(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$$

# SECTION A

1. (a) Write down the partial differential equation describing wave motion in one dimension. Define your variables. [2]

   (b) Show that a disturbance of the form $\psi(x,t) = A\cos(kx-\omega t)$ satisfies this equation. [3]

   (c) What is the *most general* form of a disturbance that will satisfy the wave equation ? [2]

2. (a) A uniform string under tension $T$ has a mass $m$, diameter $d$ and length $L$. At what speed would a wave disturbance travel down the string, in terms of these variables? [2]

   (b) The six strings of a guitar are of the same density and length and are under nearly the same tension, but have different thicknesses. On which string do waves travel the fastest? Briefly explain your answer. [2]

   (c) While a guitar string is vibrating, you gently touch the midpoint of the string to ensure that the string does not vibrate at that point. Briefly explain which normal modes cannot be present on the string while you are touching it in this way? [2]

3. (a) State Huygens' principle and draw a diagram which illustrates how it can be used to show how a circular wavefront develops with time. [4]

   (b) State Fermat's theorem and explain qualitatively how it can be used to show that at the boundary between two media of different refractive indices $n_i$ a light ray refracts (a detailed derivation of Snell's law is not required). [4]

4. (a) List three methods of obtaining linearly polarised light. [3]

   (b) Describe what happens if a light ray travelling in the $z$ direction but polarised along the $x$-axis is added to a light ray of the same amplitude travelling in the same direction but polarised along the $y$ axis and $\pi/2$ out of phase with the first. [4]

5. (a) Draw and label a diagram of a Michelson Interferometer, including light rays. [4]

   (b) Explain what is meant by the chromatic resolving power of the Interferometer, and what feature of the Interferometer can maximise it. [3]

6. (a) The diffraction pattern produced by a grating with $N$ slits is given by

$$I = I_0\left(\frac{\sin(N\beta\sin\theta)}{\beta\sin\theta}\right)^2,$$

where $\beta = (\pi/\lambda)b$ and $\lambda$ is the wavelength of the incident light and $b$ is the spacing between the slits. For the case $N = 5$ plot $I$ against $\sin\theta$ commenting on the position of all maxima and minima in intensity, and the intensity of the maxima. [5]

# SECTION B

7. (a) At a music festival in 2009, KISS were reportedly recorded with a sound level of $\beta_2 = 136$ dB at the sound tent. What is the ratio of the sound intensity of the band at that spot to the sound intensity of a jet engine 100m away, operating at sound level $\beta_1 = 110$ dB ? [6]

   (b) A source moving at speed $v_s$ towards a stationary observer emits sound waves at a frequency $f$. Show that the observer measures a frequency

$$f' = f\left(\frac{1}{1 - v_s/v}\right)$$

   where $v$ is the speed of sound in air. [6]

   (c) A rocket moves at a speed of 150 m/s directly towards a stationary observer (through stationary air) while emitting sound waves at frequency f=1100 Hz. Take the speed of sound in air to be 343 m/s.

      i. What frequency $f'$ is measured by the observer ? [1]

      ii. The observer now climbs into a car, and drives towards the rocket at 50 m/s. What frequency does the observer now measure ? [3]

      iii. Some of the sound reflects back off the car to the rocket as an echo. What frequency $f''$ does a detector on the rocket detect for the echo? [4]

8. There are two types of organ pipes known as diapasons: one is open at both ends (and is known as an *open* diapason) while the other is closed at one end and open at the other (known as a *stopped* diapason).

   (a) Write down the mathematical form of the boundary conditions on both a *stopped* diapason pipe and an *open* diapason pipe, and explain the physics giving rise to these conditions [8]

   (b) Give a formula for the allowed wavelengths in a *stopped* diapason pipe [4]

   (c) How will the formula change for an *open* diapason ? [2]

   (d) On a winter's day, the lowest note on an *open* diapason is measured to have a frequency of 64Hz. Find the length of the pipe, if the bulk modulus of air is measured as $1.42\times10^5$ N/m$^2$ and the density is measured as 1.29 kg/m$^3$. [6]

9. (a) Two sinusoidal waves travelling along a string with equal amplitudes A and with periods of $\tau_1 = 39s$ and $\tau_2 = 40s$, displacements $y_1$ and $y_2$, and angular frequencies $\omega_1$ and $\omega_2$, respectively, each have a maximum displacement at $x = 0$ and $t = 0$.

i. Show that the sum of the two waves at $x = 0$ can be written as [4]

$$y_1 + y_2 = 2A\cos\left(\frac{\omega_1 + \omega_2}{2}t\right)\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)$$

ii. For this resultant, evaluate the period of the carrier wave, $\tau_{\text{carrier}}$, and envelope, $\tau_{\text{envelope}}$. [4]

iii. After how many seconds will the resultant displacement be zero at $x = 0$? [2]

(b) For deep-water waves, the precise relationship between angular frequency $\omega$ and wavenumber $k$ is

$$\omega = \sqrt{\frac{gk}{2}}$$

where $g$ is the acceleration due to gravity.

i. What is such a functional relationship between frequency and wavenumber known as? [2]

ii. Describe what is meant by the phase velocity ($v_p$) and group velocity ($v_g$) of waves. [2]

iii. Show that the group velocity is one-half the phase velocity for deep-water waves. [3]

iv. In general, show that $v_g$ can be written as [3]

$$v_g = v_p + k\frac{dv_p}{dk}$$

10. (a) Explain what is meant by the terms

i. the refractive index of a medium, [1]

ii. the optical path length within a medium, [1]

iii. the optical path difference between two light rays. [2]

(b) What are the changes in phase when a light wave is reflected at the interface between two media if

i. the medium behind the boundary is of greater refractive index, [1]

ii. the medium behind the boundary is of lower refractive index. [1]

(c) Newton's rings are created using a large lens which is a section of a sphere with radius $R$ with one curved side and one flat side. This is placed with curved side downwards on a very flat reflecting surface so that the flat side of the lens is parallel to the reflecting surface and the point at which the lens and surface touch is designated to be the origin $O$.

i. Draw the arrangement and show that at some point $P$ distance $r$ from $O$ the vertical distance $t$ between the reflecting surface and the curved surface of the lens is given by

$$t = \frac{r^2}{2R}$$

if one assumes $t \ll R$. [5]

ii. Newton's rings are created from constructive interference of light of wavelength $\lambda$ incident from directly above the apparatus where one component is reflected from the curved surface of the lens, and the other from the flat reflecting surface. Assuming a vacuum between the lens and surface, use the optical path difference and phase changes at reflection to show that bright rings are created when

$$r^2 = (p + 1/2)R\lambda,$$

where $p$ is an integer. [4]

iii. How many rings will be created if $\lambda$ = 500nm, $R$ = 1m and at the edge of the lens $t$ = 0.1mm? How will the answer change if oil of refractive index $n_{\text{oil}} = 1.4$ is introduced between the lens and reflecting surface? [5]

11. (a) i. State the mirror/lens equation in terms of $p$ the object distance, $q$ the image distance and the focal length $f$. [2]

ii. Use this equation to prove that the image formed by a convex mirror is behind the mirror and reduced in size. Draw a ray diagram verifying this and showing that the image is upright. [5]

(b) i. Explain what is meant by the near point and far point for the eye. [2]

ii. Describe what is meant by farsightedness and nearsightedness. What type of lens is needed to cure each of these complaints and why? [4]

iii. A person has a distance between the lens in their eye and their retina of 2cm. The maximum focal length of their lens is 1.98cm. What is their near point (measured from the lens of their eye)? [2]

iv. What type of lens in their glasses would they need to obtain a near point at 25cm? Assuming the distance between the lens in their glasses and that in their eye is very small find the focal length of glasses needed to achieve this. [5]

# UNIVERSITY COLLEGE LONDON

# EXAMINATION FOR INTERNAL STUDENTS

| | | |
|---|---|---|
| MODULE CODE | : | **PHAS1224** |
| ASSESSMENT PATTERN | : | **PHAS1224A** |
| MODULE NAME | : | **Waves, Optics and Acoustics** |
| DATE | : | **15-May-12** |
| TIME | : | **10:00** |
| TIME ALLOWED | : | **2 Hours 30 Minutes** |

2011/12-PHAS1224A-001-EXAM-154



**TURN OVER**

**Answer ALL SIX questions in Section A and THREE questions from Section B**

The numbers in square brackets at the right-hand side of the text indicate the provisional allocation of maximum marks per question or sub-section of a question.

You may find the following constants useful.
Speed of sound in air is 343 m/s.
The acceleration due to gravity, $g = 9.8\text{m/s}^2$

# SECTION A

1. (a) In a pipe of diameter $d$ and length $L$ containing gas of density $\rho$ and bulk modulus $B$ at pressure $P$, at what speed would sound waves travel ? What is the impedance of the pipe ? [4]

   (b) Write down the wave equation for a wave on a stretched string, defining any terms you may use. [3]

2. (a) Two guitar players are playing the same note on their E strings, which are at slightly different frequencies $f_1$ and $f_2$.

   i. What is the frequency of the resulting oscillation ? [1]

   ii. What is the *beat* frequency ? [2]

   (b) If one of the guitarists is on a part of the stage moving towards the audience with velocity $v_s$, show that the frequency of the note heard by the audience is: [4]

$$f' = f\left(\frac{v}{v - v_s}\right)$$

   where $v$ is the speed of sound in air.

3. (a) What are the changes in phase when a light wave is reflected at the boundary between two media if:

   i. the medium behind the boundary is of greater refractive index?

   ii. the medium behind the boundary is of lesser refractive index? [2]

   (b) Transparent wax of refractive index $n = 1.3$ is deposited on top of a glass plate of width 1cm and refractive index $n = 1.5$. The thickness of the wax is 0.01mm at one end of the plate and tapers uniformly to zero at the other end of the plate, which is defined to be at $x = 0$. At this end the surface of the wax and of the glass form a small angle $\alpha$. Find the value of $\alpha$. Light is incident on the plate from above, i.e. it goes through the wax, and is normal to the surface of the glass plate. Fringes are formed due to interference of light reflected from the top surface of the wax and from the glass. At what values of $x$ do bright fringes occur if $\lambda = 520\text{nm}$? [4]

4. (a) The intensity of light viewed at the centre of a screen of a Michelson interferometer in vacuum is $I = 4I_0 \sin^2((2\pi/\lambda)d)$, where $\lambda$ is the wavelength of the light and $d$ the difference between the lengths of the two arms of the interferometer. Explain why the intensity changes if an object of refractive index $n$ and thickness $t$ is introduced into one arm. If $n = 1.002$, $t = 1 \times 10^{-4}$m and $\lambda = 500$nm, and the intensity was originally at its maximum value, what is the intensity after the introduction of the object? [3]

   (b) Outline very briefly how the interference fringe pattern is obtained by a Fabry Perot Etalon (no diagram is required), and what feature enables it to obtain sharper fringes than a Michelson interferometer. [3]

5. (a) A diffraction grating with $N$ slits each separated by distance $d$ gives a diffraction pattern with intensity

$$I = I_0 \frac{\sin^2(N(\pi/\lambda)d\sin(\theta))}{\sin^2((\pi/\lambda)d\sin(\theta))}.$$

   At what positive values of $\theta$ are the primary maxima found? [2]

   (b) If $N = 500$ and the grating width is 2mm how does the number of observable primary maxima vary across the visible spectrum, e.g. for red and blue light? [4]

6. (a) A converging lens has a focal length of 5mm. An object is placed 5.2mm to the left of the lens. Where is the image formed? [2]

   (b) A second converging lens with focal length 20mm is placed to the right of the first lens. What is the distance between the lenses if the image formed is 250mm to the left of the second lens? [2]

   (c) Draw the ray diagram showing the lenses, and the image formed by each lens. What is the lateral magnification of the system of lenses? What optical instrument could this be? [4]

# SECTION B

7. (a) The Doppler effect is demonstrated by tying a 500 Hz sound generator to a rope 1.2 m long, and whirling it around in a horizontal circle at 60 rpm. What is the difference between the highest and lowest frequencies heard by a student in the classroom ? [6]

(b) i. If you are given two sound intensities, $I_1$ and $I_2$, how is the difference in the sound levels $\beta_1$ and $\beta_2$ related to these intensities? [3]

ii. By what factor has the intensity of a sound increased if the sound level has risen by 10 dB ? [2]

(c) Two loudspeakers, at the same height, are 2.0 m apart and in phase with each other. Both emit 705 Hz sound waves into a room. A listener stands 5.0 m in front of the loudspeakers and 2.0 m to one side of their centre.

i. Is the interference at this point constructive, destructive, or something in between ? Be sure to show your reasoning clearly. [Hint: consider the distance to the listener and the wavelength of the sound waves.] [6]

ii. How would the situation differ if the outputs of the loudspeakers were $\pi$ out of phase ? [3]

8. (a) Define the phase velocity and group velocity of waves in terms of the angular frequency and wavenumber. [2]

(b) In what kind of environment are these equal ? Give an example. [2]

(c) Sketch a wavepacket and indicate which part travels with the group velocity and which part travels with the phase velocity, respectively. [3]

(d) In deep water, it can be shown that $\omega = \sqrt{gk/2}$

i. Find the relationship between group velocity and phase velocity for deep water waves. [2]

ii. Orcas (killer whales) have been filmed generating waves in deep water to knock seals off ice floes. Assume that an orca can generate a wavepacket which is simply half a sine wave (i.e. $\sin(kx)$ from $kx = 0$ to $kx = \pi$). Show that the kinetic energy *density* in this wave can be written:

$$U = \frac{1}{32}\rho g \frac{A\lambda}{\pi^2}$$

where $A$ is the wave amplitude and $\lambda$ is the wavelength. [Hint: calculate the mass of water in the wavepacket produced by an orca tail of width $l$ by integrating, and then find the kinetic energy; note that we are neglecting any fluid properties such as surface tension.] [6]

iii. The tail of an orca is around 2m wide, and sea water has density 1,025 kg/m$^3$. If the orca can move its tail up and down through a total vertical distance of 2m in 2s, calculate the energy present in a wavepacket generated by three orcas swimming side-by-side. If a Weddell seal has a mass of around 500kg, how fast would it move if the kinetic energy in the wavepacket is translated entirely into kinetic energy of the seal ? [3]

9. (a) Give a *mathematical* definition for the boundary conditions at the ends of a fixed string, length $L$. [4]

(b) If a wave on the string (mass $m$ under tension $T$) can be written as:

$$\psi(x,t) = 2A\cos(\omega t)\sin(kx)$$

i. Define $k$ and $\omega$ in terms of the given properties of the string [2]

ii. Show that the resonant frequencies $f_n$ are given by $f_n = (v/2L)n$ for $n = 1, 2, 3, \ldots$ where $v$ is the speed of the wave. [4]

(c) A lift cable has mass per unit length 4 kg/m. The lift has mass of 920 kg (assume that the tension in the cable is provided solely by the weight it supports). When the lift stops at the 3rd floor, the distance from the top of the lift to the top of the cable is 4.6m. Assuming that people weigh (on average) 80kg and act as vibrational sources at 100Hz (from the motion of their feet), how many people would need to enter the lift to excite the 15th harmonic in the lift cable (to the nearest person)? [4]

(d) Vuvuzelas were used extensively in a recent sporting event, and can be modelled as a cylinder 60cm long, open at both ends. If a small child emits a shriek into a vuvuzela starting at 500Hz and ending at 2000Hz, at what frequencies will resonance occur ? [4]

(e) Sketch the standing wave formed in the vuvuzela at the lowest frequency you have just found. [2]

10. (a) State Snell's law for angles of refraction at the surface between two media of different refractive index, defining your variables. [2]

(b) Two media of refractive index $n_i = 1.5$ and $n_r = 1.33$ are separated by a plane boundary. Light is incident on the boundary from the medium with refractive index $n_i$ at an angle of 55° to the normal to the boundary surface. What is the angle of the refracted beam? What happens if the angle of incidence is instead 75°? Would the same phenomenon be possible if $n_i = 1.2$ for the same value of $n_r$? [4]

(c) Explain what is meant by the statement that a light wave is linearly polarised. If the wave is travelling along the $z$ axis how many independent types of polarisation are there? Give two examples of linearly independent polarisations. [4]

(d) When incident on a boundary, in general the incident wave can have field components both perpendicular to and parallel to the plane of incidence. The reflection coefficient for light parallel to the plane of incidence is

$$r_{\parallel} = \frac{n_r \cos\theta_i - n_i \cos\theta_r}{n_r \cos\theta_i + n_i \cos\theta_r} \equiv \frac{\tan(\theta_i - \theta_r)}{\tan(\theta_i + \theta_r)}$$

What value must $(\theta_i + \theta_r)$ take for the reflection coefficient to be zero? [1]

(e) Use this value and Snell's law to show that $r_{\parallel} = 0$ if the angle of incidence is $\theta_i = \tan^{-1}(n_r/n_i)$, i.e. Brewster's angle. [3]

(f) Make a rough plot of the intensity of reflected light with polarisation parallel to the plane of incidence in the two cases $n_i = 1.5, n_r = 1.33$ and $n_i = 1.2, n_r = 1.33$ and mark Brewster's angle and the critical angle, and intensity for $\theta_i = 0°$ and 90° in each case. [6]

11. (a) Light is normally incident on an opaque plate with two thin transparent slits separated by distance $d$ and then viewed on a screen at distance $L$ from the board where $L \gg d$. The point $O$ on the screen is opposite the point midway between the slits, and point $P$ on the screen is distance $y$ from point $O$ with the line between the two being parallel to that separating the slits. Show that the distance travelled by the light wave from the slit closer to $P$ is $r_1 = \sqrt{L^2 + (y - d/2)^2}$ and find a similar expression for the distance from the further slit $r_2$. By making an expansion of these expressions show that in the limit that both $y$ and $d$ are $\ll L$ the distance travelled by these two light waves differs by $yd/L$. [5]

(b) The interference pattern formed is given by $I(\theta) = 4I_0 \cos^2(\pi \sin\theta d/\lambda)$, where $\theta$ is the angle from the normal to the line from the centre of the slits to point $P$. For $d = 1\times10^{-5}$ m and $\lambda = 500$nm plot the intensity pattern as a function of $\theta$ for small $\theta$. How would this change if a block of material of thickness 500nm and refractive index $n = 1.5$ were placed over one slit (without altering the intensity from that slit). [3]

(c) Show qualitatively what would happen if instead the block had no effect on phase but reduced the intensity from one slit by a half. [2]

(d) If instead the aperture in the plate were one single slit of width $a$ the intensity on the distant screen would be

$$I(\theta) = I_0 \frac{\sin^2((\pi/\lambda)a\sin\theta)}{((\pi/\lambda)a\sin\theta)^2}.$$

If an image is formed by an additional light source at a small angle $\phi$ to the normal to the plate and screen this forms the same pattern, but with $\theta$ replaced by $\theta - \phi$. If $\phi = \lambda/a \ll 1$ plot the intensity patterns formed by each source. [4]

(e) Model the pupil of the eye using this single slit and ignore any issues to do with the pupil being a circle rather than a slit. If the eye is able to notice changes in intensity of about 20% show that it can just distinguish two objects separated by an angle of 0.00015 radians if $\lambda = 500$nm and the diameter of the pupil is 4mm. (Hint: consider the intensity halfway between the peaks of the two images.) [4]

(f) Estimate the smallest letters than can possibly be read in an eye test if one stands 5m from the chart on which the letters appear. [2]