UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE	:	**PHAS2201**
ASSESSMENT PATTERN	:	**PHAS2201A**
MODULE NAME	:	**Electricity and Magnetism**
DATE	:	**12-May-09**
TIME	:	**10:00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

2008/09-PHAS2201A-001-EXAM-174

Answer ALL SIX questions from Section A and THREE questions from Section B.
The numbers in square brackets on the right-hand side indicate the provisional allocations of maximum marks per sub-section of a question.

electron charge, $e = 1.60 \times 10^{-19}$ C
permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}
permeability of free space, $\mu_0 = 1.26 \times 10^{-6}$ Hm^{-1}

SECTION A

[Part marks]

1. (a) Define the electric field **E**. [2]

(b) State Gauss' law for electrostatics in integral form. [2]

(c) By using this law, determine the electric field **E** *inside* an insulating sphere, at a distance r from the centre, assuming a uniform charge density ρ. [4]

2. (a) What is the electric field inside a conductor at electrostatic equilibrium? Briefly explain your answer. [2]

(b) If an isolated conductor carries a net charge, where is the charge distributed? [1]

(c) Two parallel metal plates of cross-sectional area A and separation d are charged to $+Q$ and $-Q$ in a vacuum. Determine the capacitance of the system. [4]

3. The potential of an electric dipole is approximated by

$$\varphi = \frac{qd\cos\theta}{4\pi\varepsilon_0 r^2}$$

(a) Define the symbols q, d, r and θ used in this expression. [2]

(b) For what values of r is this approximation justified? [1]

(c) Derive an expression for the radial component of the electric field for a given direction from the axis of the dipole. [2]

(d) Sketch the electric field lines in the vicinity of the dipole. [2]

4. (a) Define the magnetic field **B** in terms of the Lorentz force. [2]

(b) State Ampère's law for steady currents in integral form. [2]

(c) State the Ampère-Maxwell law in differential form. [2]

5. (a) State Faraday's law of electromagnetic induction in integral form. [2]

(b) State three ways of generating an *e.m.f.* in a wire. [3]

(c) Write down an expression for the energy stored in an inductor of inductance L carrying a current I. [1]

6. (a) Write down an expression for the energy stored in an capacitor of capacitance C carrying a charge Q. [1]

(b) What is the power dissipated in an ideal capacitor? [1]

(c) What is the power dissipated in an ideal resistor? [1]

(d) Define complex impedance, impedance and phase angle for a two terminal A.C. circuit. [3]

SECTION B

7. (a) Write down a general expression for the electrostatic energy of a set of n charges, $q_1, q_2, \ldots\ q_n$ at positions $\mathbf{r}_1, \mathbf{r}_2, \ldots\ \mathbf{r}_n$ respectively, and a specific expression for the case $n = 4$. [4]

A salt crystal consists of an array of positive Na and negative Cl ions, both carrying an elementary charge of magnitude e. Assume that a small "seed" crystal consists of four ions, forming a square of side 0.25 nm, as shown in Figure 1.

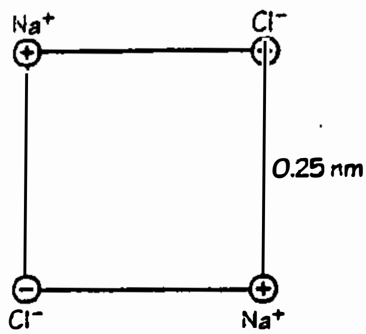

Figure 1

(b) Calculate the work done in removing one of the ions from the cluster to infinity, assuming that the other three ions remain fixed. [6]

(c) Find the electric field acting at one of the sodium ion positions due to the other ions of the cluster. [6]

(d) Hence find the force acting on one of the sodium ions due to the other ions of the cluster. [2]

(e) What is the energy density of the electric field at one of the ion positions due to the other three ions? [2]

8. The electric field inside a dielectric material is reduced by a factor ε_r, the dielectric constant, from the vacuum value. For helium and water at *s.t.p.*, $\varepsilon_r = 1.000068$ and $\varepsilon_r = 80$ respectively.

(a) Discuss briefly the mechanism responsible for the response of helium. [3]

(b) Discuss briefly the mechanism responsible for the response of water. [3]

Consider the coaxial cable of Figure 2. There are equal but opposite charges per unit length $+\lambda$ and $-\lambda$ on the two conducting elements of the cable, which acts as a capacitor.

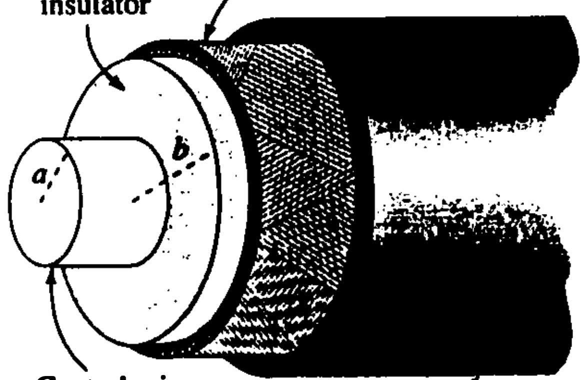

Figure 2

(c) Show that the potential difference between the conductors in the coaxial cable is given by

$$\phi = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_r}\ln\left(\frac{b}{a}\right)$$

where ε_r is the dielectric constant of the insulator inserted between the inner wire and the thin outer sheath, and calculate the capacitance per unit length and potential energy per unit length of the cable. [7]

The coaxial cable shown in Figure 2 is discharged, then used to carry a current $+I$, with uniform current density, along the inner wire and $-I$ along the thin outer sheath.

(d) Assuming that the dielectric material does not affect the magnetic field, use Ampère's law to determine the magnetic field at a distance r from the centre of the wire:

(i) for $r < a$; [4]

(ii) for $a < r < b$; and, [2]

(iii) for $r > b$ [1]

9. (a) The Biot-Savart law may be written as

$$\vec{B}(1) = \frac{\mu_0}{4\pi} \int \frac{I d\vec{s}_2 \times \hat{e}_r}{r_{12}^2}$$

Define the variables B(1), I, ds_2 and r_{12} used in this expression. [4]

(b) Find the magnetic field at the point P in Figure 3, if a current of 8 A flows in the infinitely long wire. The radius R of the semicircle is 1.2 cm. [8]

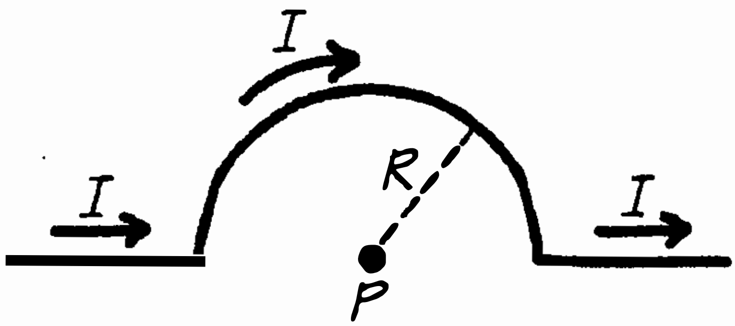

Figure 3

(c) An infinitely long wire is bent into an "L" shape and placed so that a current I flows along the y-axis towards the origin, then out from the origin along the x-axis, as shown in Figure 4.

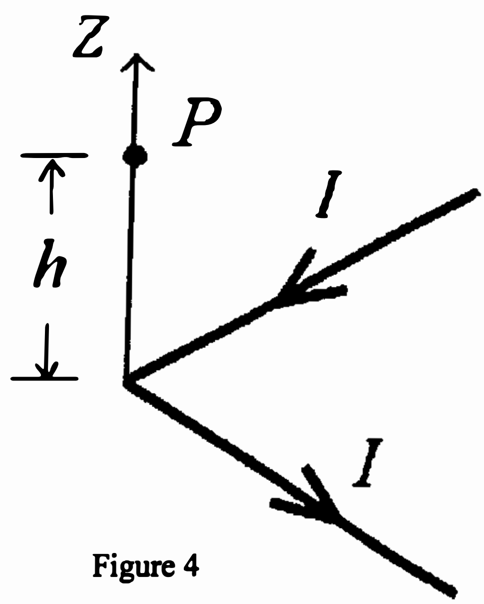

Figure 4

Calculate the magnetic field at the point P on the z-axis at a height h above the origin, and state its direction. (Ampère's law can be applied: the field of a semi-infinite wire is equal to half the field generated by the same current in an infinite wire). [8]

10. (a) Use Faraday's law to determine the *e.m.f.* induced around a coil of N turns of cross-sectional area A rotating at an angular velocity ω about an axis in the plane of the coil and normal to a uniform magnetic field B. [5]

(b) The headlight of a bicycle is powered by a small generator (dynamo) that is driven by the wheel of a bicycle. The generator is driven by means of a friction wheel, such that a point on the circumference of the friction wheel moves at the same speed as the linear speed of the bicycle. The generator contains two coils connected in series with appropriate polarity, as shown in Figure 5.

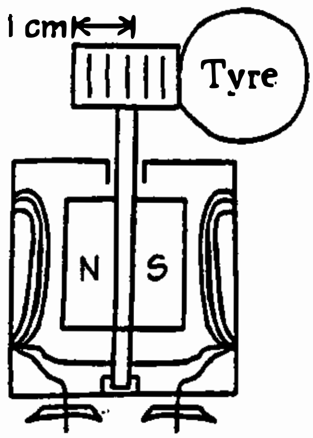

Figure 5

Each coil consists of 70 turns and has an area of 800 mm². A small permanent magnet is rotated in front of the coils, so that the magnitude of the magnetic field varies between 0.1 T and zero. If the speed of the bicycle is 5.7 ms^{-1} and the radius of the friction wheel is 10 mm, what will be the angular frequency of the friction wheel? What will be the maximum *e.m.f.*? If the resistance of each coil is 2 Ω, what is the power of the dynamo? [6]

(c) A bicycle wheel of radius R = 330 mm rotates at an angular speed ω = 38 s^{-1} in a plane perpendicular to a constant magnetic field of magnitude 0.14 T. Use Faraday's law to calculate the *e.m.f.* generated between the centre of the wheel and its rim. [5]

(d) Use the Lorentz force to calculate the *e.m.f.* generated between the centre of the wheel and its rim for the same system as described in problem (c). [4]

11. (a) State the Kichhoff Rules for electrical networks, and state the basis for these rules in terms of fundamental conservation laws. [4]

(b) Apply Kirchhoff's Rules to find the effective impedance z_{eff} of two impedances z_1 and z_2 in series. [2]

(c) The differential equation for charge in an LCR series circuit

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=0$$

is solved by

$$Q=Q_0\{\exp(-\alpha+i\omega)t\}$$

Prove that the constants α and ω are given by

$$\alpha=\frac{R}{2L},\quad \omega^2=\frac{1}{LC}-\frac{R^2}{4L^2}$$

[6]

(d) Given that the peak driving voltage in the circuit shown in Figure 6 is 110 V and the frequency of oscillation is 60 Hz, calculate the maximum current and the maximum potential drop across the inductor. [4]

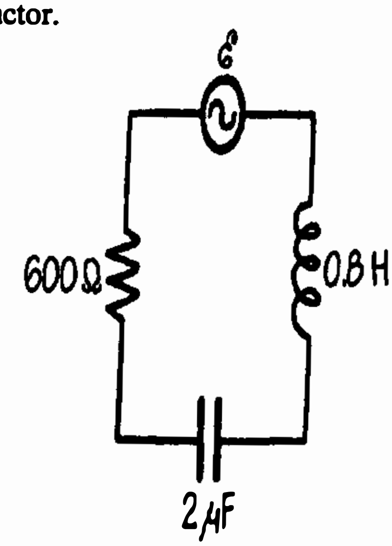

Figure 6

(e) Find the voltages across the inductor in the AC circuit of Figure 6 at times $t = 0$ and $t = 0.001$ s if the applied *e.m.f.* is given by $V = V_0\sin\omega t$. [4]

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MODULE CODE : **PHAS2201**

ASSESSMENT PATTERN : **PHAS2201A**

MODULE NAME : **Electricity and Magnetism**

DATE : **11-May-10**

TIME : **14:30**

TIME ALLOWED : **2 Hours 30 Minutes**

TURN OVER

Answer *ALL* SIX questions from Section A and THREE questions from Section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

permittivity of free space,	ϵ_0	=	8.85×10^{-12} Fm^{-1}
permeability of free space,	μ_0	=	$4\pi \times 10^{-7}$ TmA^{-1}
gravity acceleration	g	=	$9.8\ m/s^2$

SECTION A

[Part marks]

1. A charge of 1 μC is placed in position (3,4) of a cartesian system. (one unit equals one metre) Calculate:
   (a) the electric field at the origin. [2]
   (b) the x and y components of this field. [2]
   (c) the electric potential at the origin (assuming it to be zero at infinity). [2]
   (d) the electric potential at the origin when an additional charge of 1μC is placed in position (-3,-4). [2]

2. A parallel-plate capacitor has a surface area of 100 cm^2 and a plate distance of 1 cm. The charge on each plate is 10^{-9} C.
   (a) Calculate the capacitance. [2]
   (b) Sketch the electric field inside and outside the capacitor. [2]
   (c) Calculate the component of the electric field parallel and perpendicular to the plates. [2]
   (d) calculate the potential as a function of the distance to one plate, where the potential is assumed to be zero. [2]

3. Dielectrics
   (a) If a dielectric with relative permittivity $\epsilon_r = 3$ completely fills the plates of a capacitor with the same characteristics as the one of question 2, calculate again the capacitance of the capacitor and electric field between the plates. [2]
   (b) calculate the capacitance and electric field if this dielectric is only inserted into half of the capacitor (the thickness of the dielectric being the same as the plate separation, but it now only covers just half of the capacitor's surface area). [4]

4. A solenoid is 10 cm long, has a radius of 1 cm and is made with 100 loops. A current of 1 A circulates in it.

   (a) Calculate the magnetic field inside the solenoid. [2]

   (b) Sketch the magnetic field lines. [2]

   (c) How would the above considerations change if the radius was 100 cm? [2]

5. Maxwell equations

   (a) Which equation is equivalent to stating that there are no isolated static magnetic charges, and why? [2]

   (b) Write the Ampere-Maxwell equation and explain why the second term is important and needed in the case of time-dependent fields. [4]

6. Speed of light

   (a) What is the speed of light in vacuum as a function of ϵ_0 and μ_0? [3]

   (b) What is the speed of light in a transparent medium with relative dielectric permittivity $\epsilon_r = 3$ (and $\mu_r = 1$)? [3]

SECTION B

[Part marks]

7. A capacitor is composed of square plates with side l, oriented vertically in the xz plane, and separated by a distance d. A dielectric with the same dimensions and thickness d, and with dielectric constant ϵ_r can slide without friction in and out of the plates, along the z axis. Assuming the capacitor is connected to a potential difference of V,

   (a) Calculate the value of the capacitance as a function of the z position of the dielectric inside the capacitor. [3]

   (b) For which mass of the dielectric does the electric force exactly compensate gravity? [5]

   (c) If there is no gravitational field, what is the acceleration felt by the dielectric if it has the mass as derived in part (b)? [4]

   (d) How does the acceleration depend on the dielectric position if instead of constant voltage the capacitor has a constant charge Q? [8]

8. A particle with momentum p and mass m is emitted at the origin of a coordinate system along the direction $(0, 1, 1)$. A magnetic field B is aligned in the direction of the y axis.

   (a) What is the force acting on the particle, in magnitude and direction? [5]

   (b) What is the radius of curvature of the trajectory in the xz plane? [5]

   (c) Which is the value of y the particle will have after a full circle, i.e. when its (x,z) coordinates will again be (0,0) ? [5]

   (d) After this point the particle enters a region where the value of the magnetic field doubles, with the same direction. For which value of y will the particle pass for a third time through the (0,0) point in the (x,z) coordinates? [5]

9. A metallic pendulum has length l, with a mass m attached to it. It is oscillating in the xz plane, with a magnetic field B along the y axis.

   (a) Describe the motion of the pendulum if its two ends are not electrically connected. [2]

   (b) Write down the period of small oscillations if the two ends of the pendulum are not electrically connected. [3]

   (c) Describe the maximum potential difference between the two ends if the maximum oscillation angle is 10°. [8]

   (d) Describe the motion if the two ends are connected through a resistance R. [7]

10. Two infinitely long wires are placed on the floor (xy plane) on the straight lines defined as x=-1cm and x=1cm. Calculate the magnetic field along the z direction at a generic point of the plane (0,y,z) if

   (a) There is a current of 1 A flowing in the positive y direction along the wire at x = 1 cm. [3]

   (b) In both wires a 1 A current flows along the positive y direction. [3]

   (c) in the first wire a 1 A current flows along the positive y direction and in the second wire a 1 A current flows along the negative y direction. [3]

   (d) in the first wire a 2 A current flows along the positive y direction and in the second wire a 1A current flows along the negative y direction. [4]

   (e) in the case of part (b) (two parallel 1 A currents on the two wires), what current must pass along an infinitely long third wire of linear mass density 1 g/m such that it is in equilibrium with the gravitational field at the position x=0, z=1 cm (with z being the vertical axis)? [7]

11. Potentials

   (a) What is the value of the scalar potential Φ at infinity normally used in the case of no moving fields? [2]

   (b) Define the vector potential **A** and the condition normally imposed on its divergence in the case of no moving fields. [6]

   (c) Describe the relationship between the scalar potential Φ, the electric field and the vector potential in the case of time-dependent magnetic fields. [5]

   (d) Describe the relationship between the vector potential **A** and the scalar potential in the case of time-dependent electrical fields, and why it can simplify a unified description of the Maxwell equations. [7]

 END OF PAPER

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EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : **PHAS2201**

ASSESSMENT PATTERN : **PHAS2201A**

MODULE NAME : **Electricity and Magnetism**

DATE : **13-May-11**

TIME : **10:00**

TIME ALLOWED : **2 Hours 30 Minutes**

2010/11-PHAS2201A-001-EXAM-138

TURN OVER

Answer <u>*ALL*</u> SIX questions from Section A and THREE questions from Section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

SECTION A

[Part marks]

1. (a) Explain, briefly, what a hysteresis cycle is [2]

   (b) Draw a graph of a typical hysteresis cycle, and explain what happens during the different parts of the cycle. [4]

2. Consider a circuit, composed of a resistance R and an inductance L in series with an AC generator providing a voltage $V(t) = V_0 \cos(\omega t)$.

   (a) Draw a sketch of the circuit. [2]

   (b) Write the differential equation relating voltage and current in the circuit. [3]

   (c) Calculate the total impedance of the circuit, and the phase difference between its resistive and inductive components. [3]

3. A wire of length L and negligible transverse dimensions, made of an insulating material, is placed on the x axis between the origin and the point (L,0). The wire has a uniform line charge density λ.

   (a) Calculate the total charge of the wire. [2]

   (b) Using Gauss' theorem and exploiting the cylindrical symmetry of the system, show that the electric field at the point $(x, y) = (L/2, L/100)$ is

$$E = \frac{100\lambda}{2\pi\epsilon_0 L}.$$

[4]

4. Write Maxwell's equations for electromagnetism in differential form for the following two cases:

   (a) In vacuum, using the vectors **E** and **B** only. [4]

   (b) In matter, for a linear material, using the vectors **D** and **H**. [4]

5. (a) Write the generic expression of Faraday's law in integral form. [2]

   (b) A square circuit with side L lies in the $x-y$ plane, is centred on the origin and has sides parallel to the x and y axes. At time $t = 0$ it starts turning about the x axis, in a region characterised by a uniform magnetic field B parallel to z, with uniform angular speed ω. By considering the angle between the surface normal and the field, calculate the potential difference (electromotive force) induced in the circuit as a function of time. [4]

6. A particle with charge q and mass m is moving with velocity $\mathbf{v}$ perpendicular to a uniform magnetic field $\mathbf{B}$.

   (a) What is the magnitude of the Lorentz force acting on this particle? [2]

   (b) What is the radius of curvature of the path taken by the particle, and the time it takes to make a full turn? [4]

SECTION B [Part marks]

7. (a) Explain what a mirror charge is for a single point charge q in the vicinity of an infinite conducting plane. [2]

(b) Two point charges $+q$ and $-q$ are placed respectively in positions ($-a$, 0, $+a$) and ($-a$, 0, $-a$) of a cartesian reference frame, with a being positive and known. The half-space of positive x is fully occupied by a conductor. What are the positions and values of the mirror charges in this specific case? (draw a sketch) [4]

(c) Show that in the previous set-up the potential at a generic point $(-x, 0, 0)$ (for x positive) is zero. [4]

(d) Show that the x and y components of the electric field at a generic point ($-x$,0,0) are zero (it is simpler to use directly the expression for the field generated by a point charge than deriving the potential). [4]

(e) Consider a generic point ($-x$,0,0). Calculate the distances of this point from all charges, the total electric field generated by each of them in terms of these distances, and the magnitude of the electric field produced by each charge, real and virtual. Find the value of the z component of the total electric field. [6]

8. Consider a parallel-plate capacitor with square plates of side L and distance d ($<< L$) between them, charged with charges $\pm Q$. The plates of the capacitor are horizontal, with the lowest lying on the $x-y$ plane, and the orientation is such that their sides are parallel to the x and y axis, respectively.

(a) Calculate the capacitance, and the voltage across the two plates. [4]

(b) Consider a simple pendulum of length $d/2$ and mass m, hanging vertically from the centre of the top plate, that can oscillate in the $x-z$ plane. Calculate the electric field inside the capacitor, and the electrical force acting on the mass if it has charge q (the field produced by the charge q itself can be neglected, and the polarity of the capacitor is such that the force will act in the negative z direction). [4]

(c) Recall that the differential equation for a mechanical simple pendulum in the gravitational field is $ml\ddot{\theta} = -mg\theta$, where θ is the angular displacement from the vertical. Considering the electrical force only, and neglecting gravity, show that the period of small oscillations of the pendulum around its vertical axis is

$$T = 2\pi\sqrt{\frac{L^2 dm\epsilon_0}{2Qq}}.$$

[4]

(d) Show that if gravity is also accounted for, this period becomes

$$T = 2\pi\sqrt{\frac{L^2 dm\epsilon_0}{2(Qq + mg\epsilon_0 L^2)}}.$$

[4]

(e) If the direction of the electric field is reversed, for which ratio q/m does the electric force compensate gravity? [4]

9. An electric dipole is placed in the $x-y$ plane, with the positive charge $+q$ placed in position $(+a,0)$ and the negative charge $-q$ placed in position $(-a,0)$.

(a) Calculate the potential at a generic point $\mathbf{r} = (x,y)$ of the plane, due to the positive charge only. [3]

(b) Show that the potential at a generic point (x, y) of the plane, due to both charges, is given by

$$V = \frac{q}{4\pi\epsilon_0}\frac{\sqrt{y^2+(x+a)^2}-\sqrt{y^2+(x-a)^2}}{\sqrt{[y^2+(x-a)^2][y^2+(x+a)^2]}}.$$

[3]

(c) Define as $\mathbf{r}_+$ and $\mathbf{r}_-$ the vectors connecting the point defined by $\mathbf{r}$ and the two points $(+a,0)$ and $(-a,0)$, and r_+ and r_- their magnitudes; calculate the y component of the electric field at a generic point (x, y) of the plane as a function of r_+ and r_- (it is quicker to use directly the field equation from a point-charge than using the potential). [6]

(d) Calculate the potential at $\mathbf{r}$ as a function of r_+ and r_-. [3]

(e) Define θ as $\tan^{-1}(y/x)$, the angle between the vector $\mathbf{r}$ and the x axis; for large distances, $|\mathbf{r}| >> a$ express the difference $(r_- - r_+)$ as a function of a and $\cos\theta$, and express the electrostatic potential as a function of the scalar product between the dipole vector $\mathbf{p} = (2aq, 0)$ and the vector $\mathbf{r}$. [5]

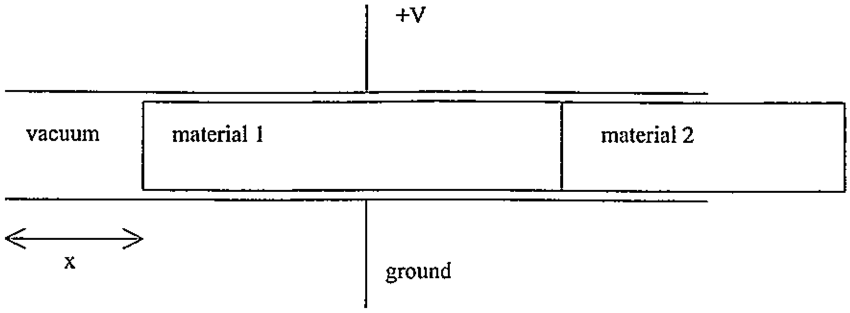

Figure 1: A parallel-plate capacitor partly filled with a dielectric

10. Consider a parallel-plate capacitor with square plates of side L and distance $d \ll L$ apart. The bottom plate lies on the $x-y$ plane, and the distance d is parallel to z. A block of dielectric material with dimensions $(L \times L \times d)$ can completely fill the space between the plates.

    (a) What would be the capacitance of the capacitor if the whole dielectric has a dielectric constant ϵ_1 ? [2]

    (b) Let us consider instead the dielectric to be composed of two materials glued together, material 1 with dielectric constant ϵ_1 and dimensions $0.6L \times L \times d$ (in x, y and z directions, respectively) and material 2 with constant ϵ_2 and dimensions $0.4L \times L \times d$ (see figure 1). The dielectric is free to move as a single block without friction along the x axis, parallel to the plates inside the capacitor, and it can also move outside the capacitor. Let us define as x the distance between the dielectric and the edge of the plate, along the x axis. A potential difference V is applied between the plates, and we can neglect the electric field outside the plates. Considering that the potential difference between two points a and b is $\Delta V = \int_a^b \mathbf{E} \cdot d\mathbf{l}$, and that $\mathbf{D} = \epsilon_0\epsilon_r\mathbf{E}$, where ϵ_r is the dielectric constant, calculate the value of the electric field $\mathbf{E}$ and of the electric displacement field $\mathbf{D}$ in the region between the plates, for the three possible regions where the space is occupied by the material 1, the one with material 2, and the vacuum. [8]

    (c) By considering the volume filled, show that the capacitance for $x < 0.4L$ can be written as

$$C = \epsilon_0 L[\frac{x + 0.6L\epsilon_1 + (0.4L - x)\epsilon_2}{d}].$$

[6]

    (d) Calculate the total energy stored in the capacitor, as a function of x. [4]

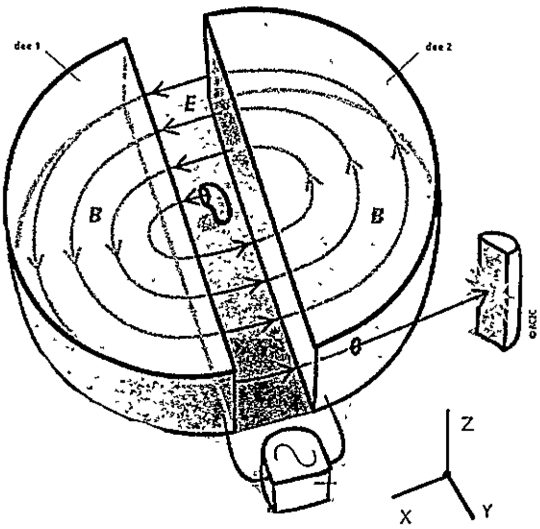

Figure 2: Sketch of a cyclotron

11. A cyclotron is a particle accelerator made of two electrodes shaped as two empty half-circles (known as dees) separated by a small gap. We consider the dees as lying on the $x-y$ plane, having radius R, both with centre in the origin, and each occupying the semiplane of positive and negative x coordinates, respectively. The cyclotron is immersed in a uniform magnetic field B parallel to the z axis (see figure 2). A particle with charge q and mass m is placed at rest halfway between the two electrodes at the origin (0,0,0). A potential difference V applied between the two electrodes then accelerates the particle while it is in the gap. The gap is so small that the influence of the magnetic field on the motion of the particle in the gap is negligible, so the magnetic field can be neglected when considering the motion of the particle in the gap.

    (a) What is the increase in kinetic energy of the particle after being accelerated by a potential $V/2$? [2]

    (b) Show that the speed of the particle after it enters the electrode on the left for the first time is $\sqrt{\frac{qV}{m}}$. [2]

    (c) Once the particle enters the first half-circle, the electric field there is negligible, and the particle proceeds in uniform circular motion due to the presence of the magnetic field. After a half-circle, the particle crosses the gap again, towards the second electrode. When this happens, the potential difference between the two electrodes is reversed, and the particle is accelerated again in the gap by the potential difference V. Show that the speed of the particle after it crosses the gap a second time is $\sqrt{3}$ times larger than the speed obtained after the first acceleration along the positive x direction. [2]

Question 11 continued on the next page

(d) After the second acceleration along the negative x axis, the particle would be entering the second(right-hand) electrode with higher speed, make a larger radius half-circle, and enter the first electrode after the voltage has been flipped again. This process can continue again and again, until the radius of curvature is larger than the radius R of the dees; then, the particle leaves the cyclotron. Show that for a generic number of half-circles, the speed of the particle increases as $\sqrt{1+2n}$, where n is the number of times the particle has traversed the gap. [4]

(e) Show that the time it takes for a particle to complete ANY half-circle is

$$T_{half} = \frac{\pi r}{v} = \frac{\pi m}{Bq}.$$

[4]

(f) Calculate the total number of times the particle can cross the gap before the radius of curvature becomes larger than the electrode's radius R. [4]

(g) Calculate the maximum speed that can be obtained in terms of the geometry of the system. [2]

END OF PAPER

UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

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MODULE NAME	:	**Electricity and Magnetism**
DATE	:	**11-May-12**
TIME	:	**14:30**
TIME ALLOWED	:	**2 Hours 30 Minutes**

2011/12-PHAS2201A-001-EXAM-172

TURN OVER

Answer ALL SIX questions from Section A and THREE questions from Section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

μ_o	$= 4\pi \times 10^{-7}$ Hm^{-1}
ε_o	$= 8.85 \times 10^{-12}$ Fm^{-1}
electron mass m_e	$= 9.11 \times 10^{-31}$ kg
proton mass m_p	$= 1.67 \times 10^{-27}$ kg
electron charge $-e$	$= -1.6 \times 10^{-19}$ C
proton charge $+e$	$= +1.6 \times 10^{-19}$ C
1 eV	$= 1.6 \times 10^{-19}$ J

Section A

1. Consider a positively charged conducting solid sphere, surrounded by a vacuum. Sketch and explain the distribution of charges, electric field and electric potential in, on and around the sphere. Explain your answer for the charge distribution by using Gauss's Law and a suitable choice of a Gaussian surface. [6]

2. An electric field in a Cartesian coordinate system (x,y,z) is given by:

$$\underline{E} = -3x^2\underline{i} - 3y^2\underline{j}$$

Show that $\underline{E}$ is a conservative field and explain what is meant by a "conservative field". Confirm that $\underline{E}$ is conservative by calculating the line integral over the closed path ABCDA , where A(0,0,0), B(1,0,0), C(1,1,0) and D(0,1,0). Sketch the path and indicate the direction of travel. [8]

3. A solar proton describes a circular orbit when it is in the magnetic field of a sunspot on the Sun. The proton has a kinetic energy of 0.2 MeV and charge q. The magnetic field B near the centre of a sunspot is 3T. Determine the radius r of the motion of the proton in the sunspot. [6]

4. Give a definition of the electrostatic potential energy of a system of point charges. A molecular ion in a vacuum is composed of 4 atoms, each carrying a charge $+q$, and each at the corner of a regular square with side length d. Show that the electrostatic potential energy U is given by:

$$U = \frac{q^2}{4\pi\varepsilon_o d}\left(4+\sqrt{2}\right) \qquad [6]$$

PLEASE TURN OVER

5. In the context of electromagnetism, briefly define what is meant by "free space". Maxwell demonstrated that in free space there is a relationship between the variation of the electric field $\underline{E}$ in space (x,y,z) and the variation with respect to time t that is given by:

$$\nabla^2 \underline{E} = \varepsilon_o \mu_o \frac{\partial^2 \underline{E}}{\partial t^2}$$

Describe briefly what is the physical interpretation of this equation for showing how electric fields can propagate in space. Explain how this equation gives us the speed of light c and give its value. [7]

6. Sketch the magnetic field lines caused by a current I flowing through a single infinitely long, straight, current-carrying wire. Briefly justify the direction of the field lines. State Ampere's Law in the integral form, and derive the magnetic field at a distance r from the wire.

Consider two infinitely long straight parallel wires, separated by a distance d and each carrying a current I flowing in the same direction. Determine the magnetic field between the wires, at a point that is distance $d/3$ from one wire and $2d/3$ from the other wire. [7]

PLEASE TURN OVER FOR SECTION B

Section B

7. Give a full derivation of the integral form of Gauss's Law of Electrostatics for an electrostatic system, but excluding dielectric materials (i.e., $\varepsilon_r = 1$). [4]

A long straight wire, with radius r_1, carries a uniform positive charge λ per unit length. It is surrounded by a long concentric hollow conducting cylinder. The cylinder has a thick wall, with an inner radius r_2 and outer radius r_3 measured from the same axis as for the wire.

a) Qualitatively describe the electric field and charge distribution within the cylinder and explain your reasoning. [5]

b) Determine the surface charge density σ on the *inner* surface of the cylinder. [6]

c) Determine the potential difference between the wire and the cylinder. [5]

8. a) State Faraday's Law of Induction and explain what is meant by self inductance for a solenoid. [2]

The magnetic field for a tightly wound long solenoid with N turns over a length l, and cross-sectional area A, carrying a current I, is given by $B = \frac{\mu_o N I}{l}$. Show that the inductance L is given by:

$$L = \frac{\mu_o N^2 A}{l}$$

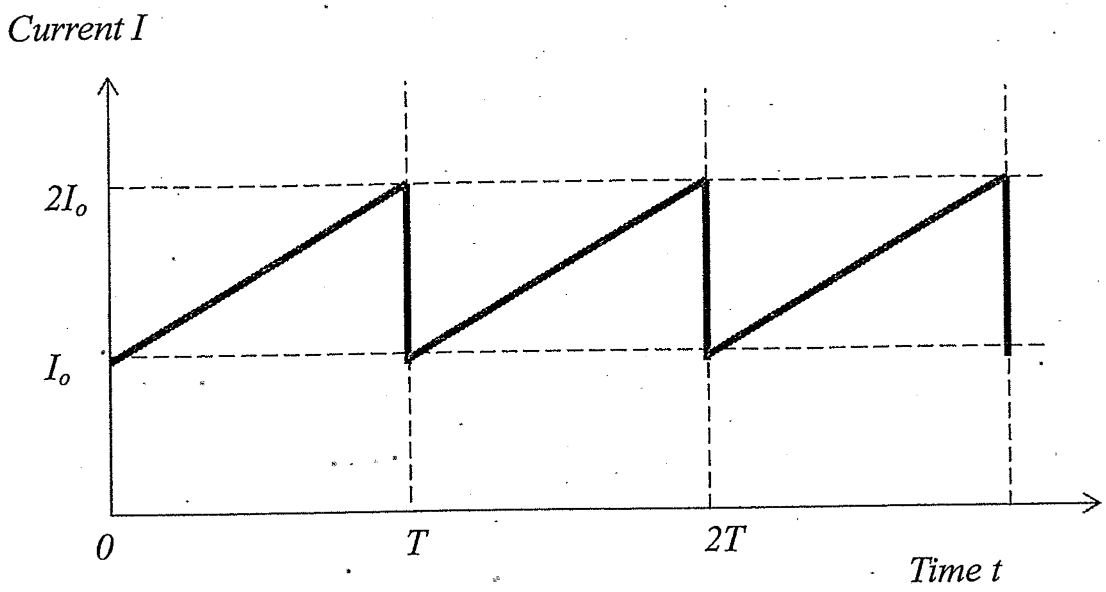
[5]

Show that when an a.c. voltage $V = V_o \sin \omega t$ is connected across an ideal inductor, the current is given by $I = I_o \sin(\omega t - \pi/2)$. Sketch the variation of V, I and the power P with respect to time t. Explain how power is stored or released during a full cycle so that there is no dissipation of electrical energy as heat, assuming that there is negligible resistance in the circuit. [7]

b) Consider another circuit containing a resistor R, in which a saw-tooth variation in the current is generated, as shown in the plot on the following page. Give an equation for the time dependence of I over one cycle, which has a period T. By determining the power lost as heat in the resistor, derive an expression for the effective current that is known as I_{rms}. [6]

(*see plot on following page*)

PLEASE TURN OVER



Plot of current against time for question 8.

9. Define the vector magnetic dipole $\underline{m}$ of a current loop of area A carrying a current I. Sketch the magnetic field lines and indicate the directions of the current, magnetic field and magnetic dipole moment. [5]

The Biot-Savart law for calculating the contribution $\underline{dB}$ to the magnetic field at a distance $\underline{r}$ from a current element $\underline{Ids}$, where I is the current and $\underline{ds}$ is an elemental length in the direction of the current, is given by:

$$\underline{dB} = \frac{\mu_o I}{4\pi\varepsilon_o r^2} \underline{ds} \times \hat{\underline{r}}$$

A circular current loop, with radius a, is positioned with its centre at the origin of a Cartesian coordinate system (0,0,0), and with its magnetic dipole moment aligned in the $+x$ direction. Use the Biot-Savart law to show that at an arbitrary point on the x-axis the magnetic field is given by:

$$\underline{B} = \frac{\mu_o \underline{m}}{2\pi(x^2 + a^2)^{3/2}}$$

[9]

The magnetic field of the Earth is similar to that of a magnetic dipole. If it is assumed that the dipole is located at the centre of the Earth, and that the radius of the equivalent current loop is much smaller than the radius of the Earth R_e, (i.e. $a \ll R_e$) estimate the size of the dipole moment of the Earth using the information below. [6]

Magnetic field at the Magnetic North Pole = $6\text{x}10^{-5}$T
Radius of the Earth (assuming a sphere) R_e=$6.4\text{x}10^3$km

PLEASE TURN OVER

10. In order to create a narrow electron beam, electrons of negligible initial velocity are accelerated by traversing a potential difference of V=10V. Calculate the final speed of the electrons v_{beam}. [2]

To investigate the effect of resistance on the speed of electrons in a conductor, now consider a copper wire with cross-sectional area A= 1mm^2 and resistance R= 0.03Ω. The wire has a voltage of V=10V applied across it. The number density of electrons in copper is N=8.5x10^{28} electrons/m^3. By determining an expression for the current and using the given information, calculate the speed of the electrons v_{wire}. [3]

Briefly describe the microscopic mechanisms underlying the difference in the speeds of the electrons in the two cases above, despite the application of the same voltage. [3]

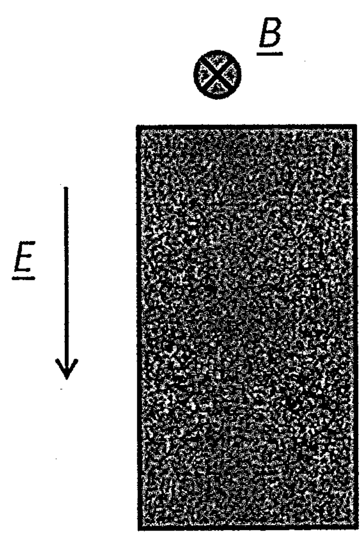
Briefly explain what is meant by the Hall Effect. [5]

A copper plate is placed in a magnetic field B and has an electric field E applied across its length. The electric field and magnetic field are at right angles as shown in the figure. Indicate the directions of the following vectors on this figure:

a) The Hall electric field E_H
b) The electron velocity v_e
c) The current flow I

Indicate the position and sign of the charges responsible for the Hall electric field.

The Hall angle θ_H is the angle between the original electric field and the Hall electric field. Calculate θ_H using v_e=0.01m/s, E=3V/m and B=1T. [7]



PLEASE TURN OVER

11. a) Briefly explain what is meant by the electrical conductivity and how it relates current density and the electric field in a conductor.

Define the electrical resistance R of a wire in terms of the electrical conductivity σ, the length of the wire l, and the cross sectional area of the wire A. Three copper wires of equal length are connected in series. They have cross-sectional areas A_1=1.0mm^2, A_2=3.0mm^2, and A_3=5.0mm^2. Calculate the voltage drop across the wire with cross-section A_3 if a total voltage of V=230V is applied across them all. [6]

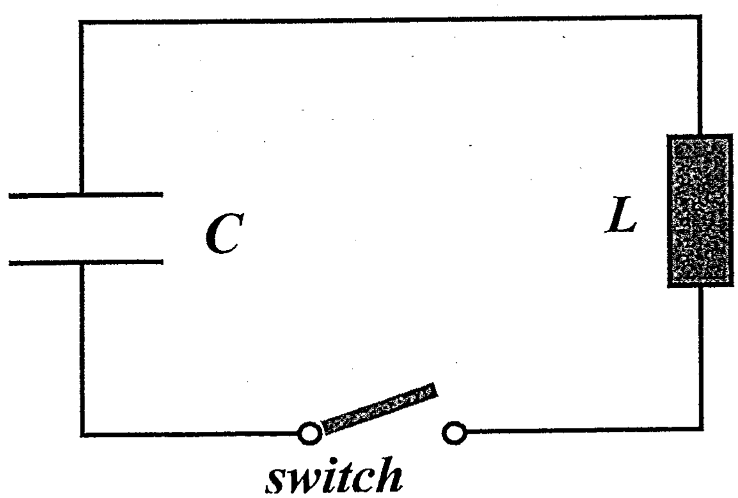
b) The circuit shown below consists of an ideal capacitor carrying a charge Q, and an ideal inductor L which are connected by wires of negligible resistance. The current in the circuit is given by I. After the switch is closed, the total energy U stored in the circuit can be expressed as:

$$U = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$

Briefly explain why the rate of change of *total* energy for this circuit is zero, i.e. $dU/dt = 0$. By differentiating U, and then substituting for I in terms of Q, show that the charge on the capacitor and the current in the circuit oscillate sinusoidally and that the angular frequency of the oscillation is given by:

$$\omega = \frac{1}{\sqrt{LC}}$$ [9]

The plates of the capacitor in this circuit initially carry charges of $\pm Q_o$. At time t=0 the switch is closed. The first time that the energy stored in the capacitor has fallen to ¾ of its initial value occurs at time t=T. Determine the inductance L in terms of T and C. [5]



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UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE	:	**PHAS2201**
ASSESSMENT PATTERN	:	**PHAS2201A**
MODULE NAME	:	**Electricity and Magnetism**
DATE	:	**10-May-13**
TIME	:	**14:30**
TIME ALLOWED	:	**2 Hours 30 Minutes**

TURN OVER

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Section A

1. Name the three classes of magnetic materials and explain briefly what determines their response when brought near a bar magnet. [8]

2. Explain why the charge on the capacitor plates is larger when a dielectric slab is placed between the two plates of a parallel plate capacitor which has a fixed voltage applied across it. State what is the new charge on the plates. [6]

3. State the equation for the electrostatic potential energy U of a system of N charges, where q_i is the charge of the i^{th} charge which is at a position r_i. Briefly explain what is meant by the electrostatic potential energy and outline in a few words how the equation is derived. [6]

4. Explain briefly why electricity is transported around the UK via high voltage transmission lines rather than low voltage, and alternating current rather than direct current. [7]

5. Electric fields are often represented as lines of force. State *three* rules for drawing electric field lines of force, using sketches to illustrate them. [6]

6. State Poisson's equation, defining all parameters, and briefly explain what is meant by the Uniqueness Theorem for electrostatics. [7]

PLEASE TURN OVER

Section B

7. Express the potential function V in terms of the electric field $\underline{E}$, which you may assume to be defined in a region of space. Also state how $\underline{E}$ can be determined if V is known. Explain what is meant by the Principle of Superposition. [6]

   A circular loop of wire of radius R is uniformly charged with a total charge Q. By determining the potential due to an elemental charge dq, derive the potential function V along the axis through the centre of the loop, in the direction perpendicular to the plane of the loop. Now determine the magnitude and direction of $\underline{E}$ from V. [8]

   Now apply the Principle of Superposition to determine $\underline{E}$ along the same axis, without making use of the potential function V. Show that this yields the same result as $\underline{E}$ determined via the relationship between V and $\underline{E}$ (as defined in the first part of this question). [6]

8. An LRC series circuit consists of an inductance L, resistance R and capacitance C that are placed in series with a source of alternating current I that has an angular frequency ω. State the equations for the equivalent resistances of the capacitor and inductor. [2]

   For each of the circuit components (i.e. the inductance, resistance and capacitance) give the phase relationship of the voltage with the current I. [3]

   Explain what is meant by a phasor diagram and how it is used to determine the magnitude and phase of the total voltage drop V across the 3 components in series. State the equation for the equivalent resistance of the LRC circuit. [7]

   Consider a circuit driven at a frequency f=50 Hz. The measured phase lag between I and V of the LRC circuit is found to be 45°. Draw a graph of how I and V vary with time t if V=0 at t=0. Determine the capacitance C if the current I=1A, resistance voltage V_R=3V and inductance voltage V_L=8V. Calculate the natural frequency ω_o of the LRC circuit if L=10mH. [8]

PLEASE TURN OVER

9. State Gauss' Law in the differential form. [1]

A static electric field has the form

$$\underline{E}$$

where K is a constant, and x, y, z are Cartesian coordinates as usual. Use the differential form of Gauss' Law to determine the charge density ρ that gives rise to this field. [2]

Consider a cube of side length L, that has one corner at the origin of the coordinates, and three adjacent sides parallel to the positive x-, y- and z- axes (as shown in Figure 1 below). Calculate the total charge Q contained within this cube due to the above derived charge density ρ. [4]

Determine expressions for the elemental area vectors $\underline{dS_i}$ that describe each of the faces of the cube (where i=1,…,6, and the direction vector for each face is shown in Figure 1 below). [3]

Now calculate the flux of $\underline{E}$ out of each face. [7]

State the integral form of Gauss' Law and show that it is satisfied for the flux determined above. [3]

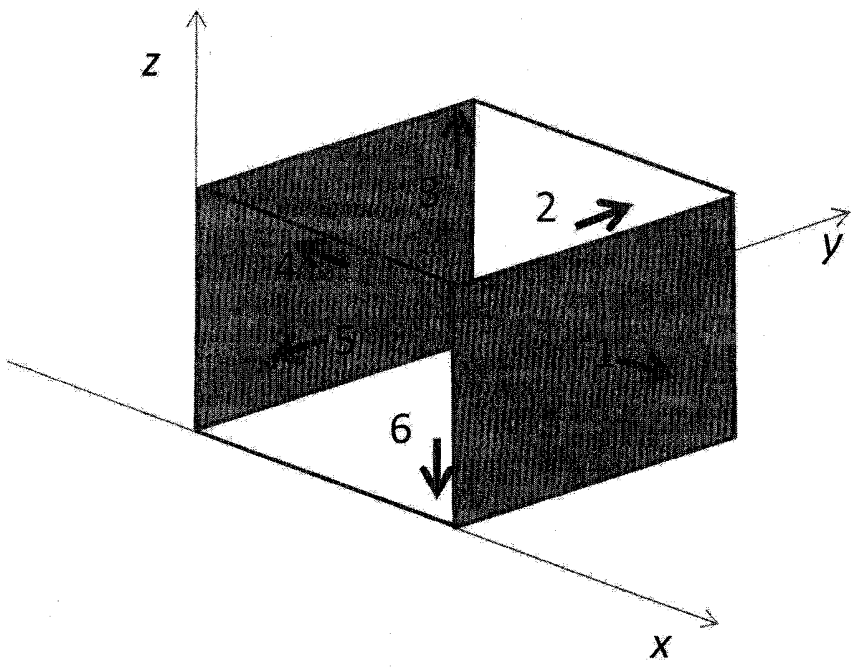

Figure 1 Cube diagram for question 9.

PLEASE TURN OVER

10. State Faraday's Law of electromagnetic induction. [2]

Consider a rectangular coil, with corners labelled consecutively a, b, c and d, that is tightly wound with N turns of wire. The coil has a short side length $da = bc = x$ and long side length $ab = cd = y$. The coil is pivoted through the middle of the short sides, and is rotating with angular speed ω in a uniform magnetic field $\underline{B}$. The coil's rotational axis is perpendicular to the direction of $\underline{B}$. Sketch and label a diagram of this configuration. [4]

Find an expression for the emf ε induced in the coil in terms of $\underline{B}$, x, y, N, ω and time t. The emf drives a current I through the coil. Indicate the direction of I on your sketch and explain your reasoning. [5]

Determine the magnitude and direction of the force on each side of the coil. [6]

By considering the potential energy of the current carrying coil, show that the power dissipated is consistent with Faraday's Law of electromagnetic induction. [3]

11. State Kirchhoff's Laws for electrical circuits and explain what they mean physically. [6]

Show how Kirchhoff's Laws can be used to determine the equivalent resistances for

(a) resistances in series, [3]

(b) resistances in parallel. [3]

For the circuit shown in Figure 2 below, determine the current I_1 in terms of the emfs ε_1 and ε_2; and the resistances R_1, R_2 and R_3: [8]

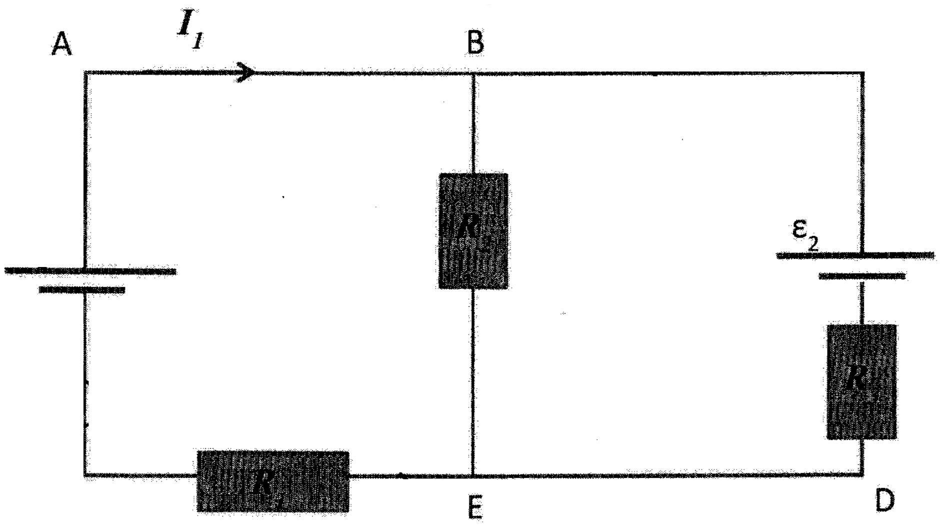

Figure 2 Electrical circuit diagram for question 11.

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