9. (a) By using the Biot-Savart law

$$\underline{dB} = \frac{\mu_0 I}{4\pi} \frac{\underline{ds} \times \hat{r}}{r^2}$$

determine the magnetic field $\underline{B}$ at the centre of a circular loop of radius R and current I. [5]

(b) A long, cylindrical conductor is solid throughout and has radius R. A current I flows through the conductor. Assume that the electric charges flow parallel to the axis of the cylinder and pass uniformly through the entire cross section. By using Ampère's law, determine the magnetic field inside the conductor at a distance r from the axis. [5]

(c) A square coil and a rectangular coil are each made from the same length of wire. Each contains a single turn. The long sides of the rectangle are twice as long as the short sides. Find the ratio $\tau_{\text{square}}/\tau_{\text{rectangle}}$ of the maximum torques that these coils experience in the same magnetic field when they carry the same current. [10]

10. (a) By using Ampère's law determine the magnetic field B generated inside an infinitely long solenoid, with current I and n turns per unit length. [6]

(b) A uniform magnetic field $\underline{B}$ is perpendicular to the plane of a circular wire loop of radius r. The magnitude of the field varies with time according to $B = B_0 \exp(-t/\tau)$, where B_0 and τ are constants. By using Faraday's law, find an expression for the EMF in the loop as a function of time. [8]

(c) The magnetic flux through each of five faces of a die (singular of "dice") is given by $\Phi_B = \pm N$ Wb, where $N = 1, ...5$ is the number of spots on the face. The flux is positive (outward) for N even and negative (inward) for N odd. What is the flux through the sixth face of the die? [6]