UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

M.Sci.

Mathematics M254: Problem-solving in Pure Mathematics

COURSE CODE	:	**MATHM254**
UNIT VALUE	:	**0.50**
DATE	:	**03-MAY-02**
TIME	:	**10.00**
TIME ALLOWED	:	**3 hours**

02-C0954-3-30

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. For each natural number n evaluate the determinant

$$\begin{vmatrix} \binom{0}{0} & \binom{1}{0} & \binom{2}{0} & \binom{3}{0} & \cdots & \binom{n}{0} \\ \binom{1}{1} & \binom{2}{1} & \binom{3}{1} & \binom{4}{1} & \cdots & \binom{n+1}{1} \\ \binom{2}{2} & \binom{3}{2} & \binom{4}{2} & \binom{5}{2} & \cdots & \binom{n+2}{2} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \binom{n}{n} & \binom{n+1}{n} & \binom{n+2}{n} & \binom{n+3}{n} & \cdots & \binom{2n}{n} \end{vmatrix}$$

2. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function differentiable at every point of $\mathbb{R}$.
   (a) Show that f' does not have to be continuous.
   (b) Show that for every $a < b$ and every c lying between $f'(a)$ and $f'(b)$ there is $a < \xi < b$ so that $f'(\xi) = c$.

3. Let $x \neq 0$ be a real number such that $\{x\} + \{\frac{1}{x}\} = 1$. For each integer n find $\{x^n\} + \{\frac{1}{x^n}\}$. (Here $\{x\}$ denotes the fractional part of x, i.e. $0 \leq \{x\} < 1$ and $x - \{x\}$ is an integer.)

4. Suppose that $g, h : [0,1] \to [0,1]$ are continuous strictly increasing functions such that $g(0) = h(0) = 0$ and $g(1) = h(1) = 1$. Show that there are $a, b \in [0,1]$ so that $g(b) - g(a) = h(b) - h(a) = 1/2$.

5. We say that $P = \{n_1, n_2, \ldots, n_k\}$ is a partition of a positive integer n if $n_1 \leqslant n_2 \leqslant \ldots \leqslant n_k$ are positive integers such that $n = n_1 + n_2 + \ldots + n_k$. A partition P of n is good if every integer $1 \leq i \leq n$ has a unique partition which consists of the elements of P only. (For instance, $\{1, 2, 2\}$ is a good partition of 5 but $\{1, 1, 2\}$ is not a good partition of 4 since $2 = 2 = 1 + 1$.)

Prove that $\{1, 1, ..., 1\}$ is the only good partition of n if and only if $n + 1$ is prime.

END OF PAPER