UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics C325: Real Fluids

COURSE CODE	:	**MATHC325**
UNIT VALUE	:	**0.50**
DATE	:	**07-MAY-04**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

04-C0917-3-40

TURN OVER

All *questions may be attempted but only marks obtained on the best* **four** *solutions will count. The use of an electronic calculator is* **not** *permitted in this examination.*

You may assume that external (body) forces are absent.

1. Write down the constitutive relation for an isotropic Newtonian fluid with constant density ρ and coefficient of viscosity μ in terms of the stress tensor, σ_{ij}, and the rate of strain tensor, e_{ij}.

   Assuming the Cauchy equations of motion,

$$\rho \frac{Du_i}{Dt} = \frac{\partial \sigma_{ij}}{\partial x_j},$$

   derive the Navier-Stokes momentum equations for an incompressible Newtonian fluid in the form

$$\rho \left[\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} \right] = -\nabla p + \mu \nabla^2 \mathbf{U},$$

   in the usual notation.

   Write down a formula for the rate of energy dissipation per unit volume, ϕ, in terms of the rate of strain tensor, e_{ij}. Calculate the dissipation function ϕ for the steady Couette flow in a planar channel of width d with one wall fixed and the other wall moving with constant speed u_0.

2. Incompressible fluid of kinematic viscosity ν is contained in a channel between two parallel walls at $y = 0$ and $y = d$ in the (x, y)-plane. The walls of the channel perform periodic oscillations in the x-direction with the speeds $u = u_0 \cos(\omega t)$ and $u = u_1 \cos(\omega t)$, at $y = 0$ and $y = d$, respectively. Here u_0, u_1 are constant and there is no additional pressure gradient along the channel. Verify that a time-periodic, uni-directional, flow is possible with the velocity profile of the form

$$u(y,t) = \mathrm{Re}\left\{ \mathrm{e}^{i\omega t} \left[u_1 \frac{\sinh(\alpha y)}{\sinh(\alpha d)} + u_0 \frac{\sinh(\alpha(d-y))}{\sinh(\alpha d)} \right] \right\}$$

with $\alpha = (1+i)\sqrt{\omega/(2\nu)}$.

Find the limiting form of the velocity profile in the case of large viscosity, $\nu \to \infty$, and give a qualitative interpretation of your result.

3. Incompressible viscous fluid with density ρ and kinematic viscosity ν flows steadily in an infinitely long planar channel between parallel walls at $y = 0$ and $y = d$ in the (x, y)-plane. A constant pressure gradient, $\partial p/\partial x = -\rho G$, is applied along the channel. The walls are made of a permeable material and the fluid is injected into the channel through the wall at $y = 0$ and extracted from the channel through the second wall at $y = d$ with the same velocity, $v = v_w$, in the y-direction. Show that a two-dimensional flow in the channel is possible with the x-component of the velocity vector of the form

$$u = \frac{G}{v_w}\left[y - d\frac{\exp(v_w y/\nu) - 1}{\exp(v_w d/\nu) - 1} \right].$$

Determine the y-component of the velocity vector in the flow.

In the case of weak injection, $v_w \to 0$, show that the flow reduces to a Poiseuille flow due to a constant pressure gradient.

Discuss briefly the case of strong suction through the lower wall, $v_w \to -\infty$, with the focus on the near-wall layer of thickness $y = O\left(|v_w|^{-1}\right)$.

4. State the assumptions of the lubrication approximation for a two-dimensional flow in a narrow gap between two solid boundaries.

Incompressible viscous fluid fills a narrow gap between a flat plate at $y = 0$ and a flexible wall at $y = h(x, t)$ in the (x, y)-plane. Assuming that the flow is governed by the lubrication approximation, show that

$$h_t = \frac{1}{12\mu}\left(h^3 p_x\right)_x$$

where $p = p(x, t)$ is the pressure in the gap.

The pressure along the gap in the range $x > 0$ varies according to

$$p = \frac{p_0}{m+1} x^{m+1}$$

with some constants p_0 and m. Show that the shape of the boundary can take a self-similar form,

$$h(x, t) = t^\alpha H(\xi), \qquad \xi = x/t^\beta$$

provided that

$$\alpha H + \frac{1+2\alpha}{m-1}\xi H' = \frac{p_0}{12\mu}\left(\xi^m H^3\right)'.$$

Derive an implicit solution, $\xi = \xi(H)$ for the case $\alpha = -1, m = 2$ when $\xi > 0$.

5. Show that the streamfunction in a two-dimensional, slow steady flow of incompressible fluid satisfies the equation

$$\nabla^2\left(\nabla^2\psi\right) = 0.$$

An axisymmetric slow flow is governed by the equation for the streamfunction of the form $D^2(D^2\psi) = 0$ where

$$D^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)$$

in spherical polar coordinates. Derive the Stokes solution for the flow past a sphere of radius a placed in a uniform stream with the speed u_∞.

You may assume the continuity equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(u_\theta\sin\theta\right) = 0.$$