UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

M.Sc. *PG Dip*

Operational Research

COURSE CODE	:	**MATHGM03**
DATE	:	**08-MAY-06**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours**

06-N0133-3-40

TURN OVER

*All questions may be attempted, but only marks obtained in the best four solutions will count. The use of an electronic calculator is **not** permitted in this examination.*

1 Let the function $f : R^2 \to R^1$ be defined by $f\left((x,y)^T\right) = x^2 - 4x + y^2 - 4y + 8$.

(i) Carry out one iteration of the method of steepest descent to minimise f using the initial point $(0,1)^T$;

(ii) Sketch the level sets of f;

(iii) What are the maximum and minimum values of f within the set $S = \{(x,y)^T : 0 \le x \le 1, 0 \le y \le 1\}$?

(iv) Show that the value of f evaluated at $(2,2)^T$ is a global minimum.

2 A doctor's surgery has N receptionists to receive telephone calls. If all of the receptionists are busy answering calls, any further calls are blocked and cleared from the system and the caller has to ring back later.

Assume that the occurrence of new calls follows a Poisson process with rate parameter λ and that service times are exponentially distributed with mean $1/\mu$. If n denotes the number of calls being dealt with, the probabilities of different events occurring in a small interval of duration h are:

$$\begin{aligned} P(\text{1 new call}) &= \lambda h + o(h) && \text{when } n < N \\ &= 0 && \text{when } n = N \end{aligned}$$

$$P(\text{1 call finishes} \mid n \text{ calls being dealt with}) = n\mu h + o(h)$$

$$\begin{aligned} &P(\text{0 calls start or finish} \mid n \text{ calls being dealt with}) \\ &\qquad = 1 - \lambda h - n\mu h + o(h) && \text{if } n < N \\ &\qquad = 1 - N\mu h + o(h) && \text{if } n = N. \end{aligned}$$

(i) Derive differential equations for the probabilities of different numbers of calls being dealt with at any one time.

(ii) Deduce the steady state distribution in terms of N and ρ, where $\rho = \lambda/\mu$.

3 (a) A large group of guests at a wedding is to be seated around a circular table. Let p_{ij} be the probability that guests i and j would find each other pleasant company (note that $p_{ij} = p_{ji}$).

A seating plan is required that maximises the probability that all guests are seated next to people that they find pleasant.

(i) Describe how the nearest neighbour algorithm can be used to solve this problem.

(ii) Can you suggest ways in which the algorithm can be improved for this problem?

(b) In Texas, days are either windy or calm. Mr Bush has two flagpoles outside his ranch. On a windy day each standing flagpole falls with a probability p. Mr Bush waits until the start of the next calm day to put up any that have fallen. A windy day is followed by another windy day with probability q. If there is a calm day, the following day is calm with probability r.

Formulate a Markov chain for this scenario with states corresponding to the day's weather and the number of flagpoles that are down at the end of the day.

4 A patient is given an intravenous infusion of a drug at a constant rate E mg/hour. The drug is eliminated according to a first order process with elimination rate constant γ per hour. The drug has an apparent volume of distribution of V litres in his body.

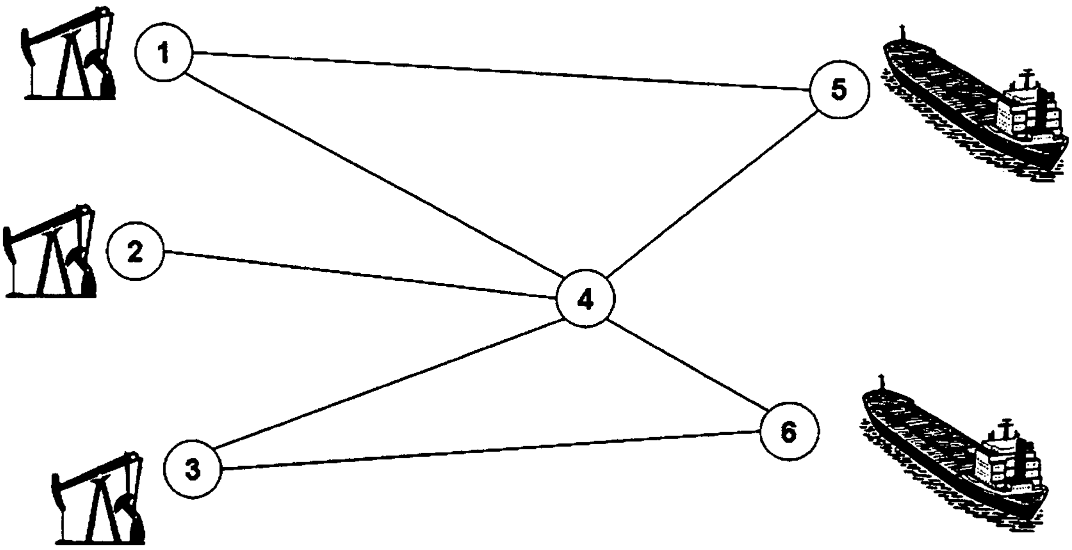
(i) If the plasma concentration of the drug at t hours after the infusion starts is $c(t)$ mg/litre, then write down a differential equation relating $c(t)$, t, E, γ and V.

(ii) If c(0) = 0, solve this equation. Hence, or otherwise, show that the steady state drug plasma concentration in mg/litre, $c_{ss} = \dfrac{E}{V\gamma}$.

(iii) The infusion is stopped after T_1 hours. After a further period of length T_2 hours the infusion is restarted and a bolus injection is administered to the patient to bring his plasma drug concentration immediately to the steady state. What dose, in mg., should be injected?

5 (a) Give an example of a linear programming problem that has more than one optimum solution while the corresponding integer programming problem has none.

(b) The diagram below represents a network of oil pipelines in a fictitious state. The oil fields at nodes 1, 2 and 3 can produce a maximum of M_1, M_2 and M_3 units of oil per day respectively. The network of pipelines shown is used to transfer this oil to the ports at nodes 5 and 6, directly or via the intermediate pumping station at the node 4. The maximum possible flow along the pipeline linking node i to node j is F_{ij} units per day. The government wants to maximise the flow of oil reaching the ports.



(i) Write down a linear programming problem which when solved could assist the government in achieving this aim.

During a period of civil war, it is feared that rebel forces will attempt to destroy sections of pipeline. To secure the pipeline linking nodes i and j from attack, the government needs to spend s_{ij} dollars per day. Pipelines that are not secured cannot be used. The government has a total budget of S dollars per day available for securing the network of pipelines and wants to maximise the flow of oil that reaches the ports within this budget.

(ii) Write down an optimisation problem which when solved could be used to determine how best to organise defences in order to maximise the total flow of oil that reaches the ports.