UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics M310: Operational Research and Mathematical Modelling

COURSE CODE	:	**MATHM310**
UNIT VALUE	:	**0.50**
DATE	:	**11-MAY-04**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

04-C0968-3-30

TURN OVER

*All questions may be attempted, but only marks obtained in the best four solutions will count. The use of an electronic calculator is **not** permitted in this examination.*

1 (a) Suppose $f: R^2 \to R^1$ is defined by the expression $f(x,y) = 2x^3 + 4y^2 - 5y$. Using the origin as an initial point, carry out one iteration of the method of steepest descent starting the search for a minimum value for f.

(b) The final day of an international golf tournament involves 10 rounds of golf, each involving a golfer from America playing against a golfer from Europe. Each team has 10 players indexed 1, . . ., 10. The American captain is so confident that he has declared in advance the order in which his golfers will play on the final day. The European captain must decide the best order in which his team should play. For $1 \leq i \leq 10$ and $1 \leq j \leq 10$, let $p_{i,j}$ denote the European captain's estimate of the probability that player i from his team would beat player j from the American team. The European captain has asked an Operational Research analyst for advice. State an integer programming problem whose solution would help the European captain to choose a playing order for his team that would maximise the expected number of wins.

2 (a) Suppose f and g are two functions given by $f(x) = x$ and $g(x) = y$, and that $h(x, y) = \max\{f(x), g(y)\}$.

(i) Sketch the level sets of h.

(ii) Hence, or otherwise, find the maximum value of h within the set S given by

$$S = \{(x,y)^T : x^2 + y^2 \leq 4\}.$$

(b) Jim has a water barrel in his garden to collect rain water off the roof, which he uses to water his vegetables. If it has not rained in the past 24 hours, he does the watering at 8 p.m., using one unit of water from the barrel. Otherwise he does not bother doing any watering, and the barrel fills up by one unit, provided it is not already full. The barrel holds up to 2 units of water and the probability that it rains in any given 24 hour period is p.

(i) Write down a Markov transition matrix for the amount of water in the barrel just after 8 p.m. each day (i.e. after Jim has done the watering on days it has not rained).

(ii) If there are two units in the barrel just after 8 p.m., what is the expected number of days before the barrel empties?

3 (a) A single server queue operates on a first-come first-served basis. Mary arrives at the queue and notices she is the nth person in the system, including the person currently being served ($n>1$). Assuming the service times are exponentially distributed with mean $1/\mu$ and that no one leaves the queue once they have joined, how long can she expect to wait before being served?

(b) Suppose, instead, that whenever the server has finished with one person, the next is picked randomly from the people currently in the queue. If there are no more arrivals after Mary, what is now her expected waiting time before being served?

4 (a) Some drugs are highly toxic if administered by bolus injection, but may be infused intravenously without any ill effects.

Give a pharmacokinetic explanation for this phenomenon.

(b) D units of a drug is administered to patients by a single bolus injection into the blood stream. It is cleared from the blood stream as a first order process with coefficient R hour^{-1}.

(i) Show that the amount of drug in the blood stream, x, at time t hours after the injection is given by, $x = De^{-Rt}$.

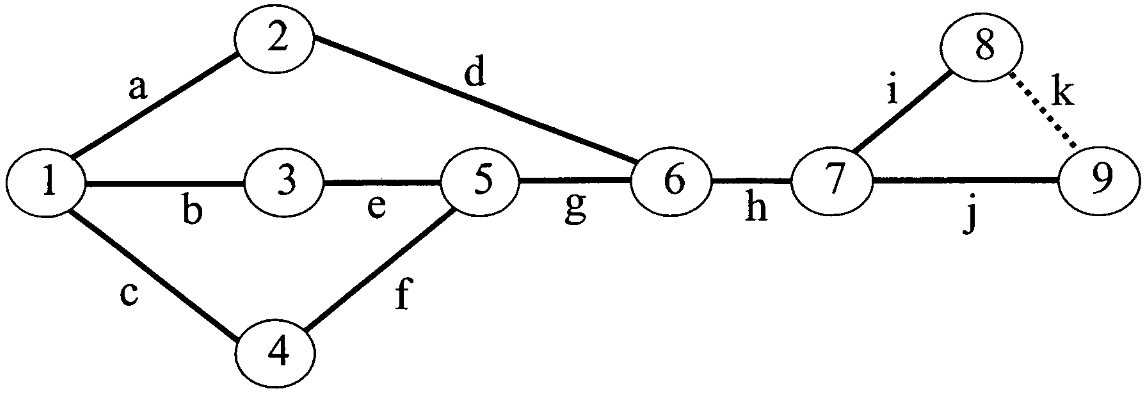
(ii) The half-life of a drug is the time taken for its concentration to fall by one half. Deduce that the half-life of the drug, $t_{1/2}$, is given by the formula:

$$t_{1/2} = \frac{\log_e 2}{R}$$

(iii) How long will it be before the amount of drug in the blood stream falls to $D/8$ units?

(c) Mrs. Green has a heart condition and is meant to take a 2-unit dose of a drug by bolus injection into the blood stream at 9 a.m. every morning. The drug has a half-life of 8 hours and a volume of distribution of 100 litres. One day she is admitted to hospital because of worsening symptoms. At 5 p.m. the concentration of the drug in the blood stream is measured to be 1.4 units ml^{-1}. Deduce whether Mrs. Green took her drug correctly that morning.

5 A firm of builders is constructing a pre-fabricated house for a client. The network diagram below shows the precedence relationships between the different tasks that comprise the project and the table gives a description of each task and its typical duration based on the firm's past experience. Task **k** is a so-called "dummy task".



Task	Description	Duration (days)
a	Construct roof at factory A.	$t_{12} = 5$
b	Construct walls at factory B.	$t_{13} = 4$
c	Lay steel rods for base of structure.	$t_{14} = 2$
d	Transport roof to site.	$t_{26} = 2$
e	Transport walls to site.	$t_{35} = 1$
f	Pour concrete onto steel rods and allow to set.	$t_{45} = 5$
g	Erect walls.	$t_{56} = 3$
h	Position roof.	$t_{67} = 1$
i	Fix tiles to roof.	$t_{78} = 2$
j	Clean interior.	$t_{79} = 1$
k	-	$t_{89} = 0$

(i) What is the role of "dummy tasks" in critical path analysis?

(ii) Based on the information in the table, calculate the minimum completion time for the project and the "slack" for activities **a**, **b** and **c**. Hence identify the critical path for this project.

The client has offered to pay extra if the building firm can complete their work in no more than ten days. The firm's manager can authorise the use of more expensive cement to reduce the duration of task **f** to 2 days. She can also authorise overtime payments at one of the factories to halve the duration of either task **a** or task **b**.

(iii) Show that it is feasible to complete the project in ten days.