

UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:–

B.Sc. M.Sci.

Mathematics B46: Mathematics and Statistics for Computer Scientists

COURSE CODE : **MATHB046**

UNIT VALUE : **0.50**

DATE : **20–MAY–04**

TIME : **10.00**

TIME ALLOWED : **2 Hours**

*There are two sections. Full marks may be obtained by answering **five** questions, but no more than three questions from a single section will count.*

Statistical tables are provided.

The use of an electronic calculator is permitted in this examination.

SECTION A: Use a separate answer book for this section

1. As part of an educational experiment, a random sample of 14 Inner London year 8 pupils are given a new software package for computer-assisted practice of mathematics at home. The following summer, these 14 pupils obtain the following percentage marks for mathematics:

63, 88, 52, 66, 73, 68, 59, 62, 91, 75, 69, 74, 72, 80.

- (a) Find the median and the upper and lower quartiles of this data set.
- (b) Write down the range and the interquartile range of these data.
- (c) Calculate the sample mean and the sample standard deviation.
- (d) The average percentage mark for mathematics obtained by the other Inner London year 8 pupils (who were not given the new software) is 66.3%. Perform an appropriate statistical hypothesis test, at a significance level of 5%, to assess whether Inner London year 8 pupils with the new software obtain on average higher marks for mathematics than Inner London year 8 pupils without the new software. Please state any assumptions that you make and formulate your conclusion clearly. State one way in which this study can be improved in order to obtain more conclusive results.

2. (Recall that an exponentially distributed random variable X with mean $E(X) = 1/\lambda$ has probability density function $f(x) = \lambda e^{-\lambda x}$ for $x > 0$; its variance is $\text{Var}(X) = 1/\lambda^2$.)

The time X until failure (hereafter called the 'failure time') of a certain type of laptop computer is exponentially distributed with a mean of 10 years.

- (a) Find the maximum guarantee time that the manufacturer of these laptops can provide, if he wants to ensure that no more than 5% of these laptops fail within their guarantee time.
 - (b) The London Bytes Shop sells 200 laptop computers of the type concerned. Assume that the failure times of the laptops are independent of each other.
 - (i) Approximately calculate the probability that the average failure time of the 200 laptops sold by The London Bytes Shop exceeds 11 years.
 - (ii) Approximately calculate the probability that more than 120 of the laptops sold by The London Bytes Shop fail within 11 years.
3. Suppose that all University of Poppleton telephone numbers consist of 6 digits, the first 3 of which are 651. Assume that all sequences of the remaining 3 digits are equally likely. Find the probabilities that a randomly selected University of Poppleton telephone number
- (a) ends with 3 identical digits;
 - (b) consists of 6 distinct digits;
 - (c) contains exactly 1 zero;
 - (d) consists of 6 distinct digits, one of which is zero;
 - (e) consists of 6 distinct digits or contains exactly 1 zero;
 - (f) contains exactly 1 zero, given that the number consists of 6 distinct digits.

4. Voice recognition software used by police detectives classifies voice recordings as either male or female. Each male voice recording is classified correctly with probability 0.95, while each female voice recording is classified correctly with probability 0.9. Suppose that each voice recording analyzed with the software originates from a man with probability 0.75 and from a woman with probability 0.25. Assume that all voice recordings analyzed are independent of each other.
- (a) Calculate the overall proportion of voices analyzed with the software that are classified incorrectly.
 - (b) Of all voice recordings that are classified as male, what proportion originated from women?
 - (c) Five independent, randomly sampled voice recordings are analyzed with the software. Denote by X the number of these voice recordings that are classified incorrectly. Name the distribution of X . Calculate $P(X = 2)$.
 - (d) Denote by Y the number of independent voice recordings analyzed with the software up to and including the first one that is classified incorrectly. Name the distribution of Y . Find $P(Y = 7)$.
 - (e) You are told that of the first 20 voice recordings analyzed, at least one was classified incorrectly. Given this information, calculate the probability that the first 5 voice recordings were all classified correctly.

Section B: Use a separate answer book for this section

5. (a) Prove, from the definition of the derivative, that the derivative of the function $y = 2x^3 - x^{-1}$ equals $6x^2 + x^{-2}$.
- (b) Define the function $\arctan(x)$, stating for which values of x your function is defined. Explain why the derivative of $\arctan(x)$ is $1/(1 + x^2)$.
- (c) Find the area which lies below the graph of the function $y = x(2 - \ln x)$ but above the x -axis.
6. (a) Find the following limits:
- (i) $\lim_{x \rightarrow 0} \frac{e^{5x} - \sin(5x) - 1}{\sin^2(2x)}$;
- (ii) $\lim_{x \rightarrow 0} \frac{2 \ln(1+x) - 2 \sin x + x^2}{\tan x - x}$;
- (iii) $\lim_{x \rightarrow 0} \frac{\int_0^x \cos^2 t dt}{\sin x}$.
- (b) Find the Taylor series expansion of $f(x) = \cos x \cos(2x)$ at the point $x = 0$.
7. (a) Find all stationary points of the function $f(x, y) = e^{x^2 - y^2}(x - 2y)$ and classify them as local maxima, local minima or saddle points.
- (b) Find the volume of the 3-dimensional region which consists of the points satisfying the following inequalities: $0 \leq x \leq \pi$, $0 \leq y \leq \pi$, $0 \leq z \leq \sin(x + y)(x + y)$.
8. Define $f(x)$ in the following way: $f(x) = x^2$ when $0 \leq x \leq \pi$, $f(x) = -x^2$ when $-\pi \leq x \leq 0$, and f is periodic with period 2π .
- (i) Draw the graph of function f .
- (ii) Find the Fourier series expansion of f .