UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Eng. *M.Eng.*

Mathematics E001: Mathematics

COURSE CODE : **MATHE001**

UNIT VALUE : **0.50**

DATE : **05-MAY-06**

TIME : **10.00**

TIME ALLOWED : **3 Hours**

06-C1030-3-200

TURN OVER

All questions may be attempted but only marks obtained on the best **seven** *solutions will count.*
The use of an electronic calculator **is** *permitted in this examination.*

1. (a) Find $\frac{dy}{dx}$ for

$$(i) \quad y = \ln\left(\frac{\cosh x}{x}\right),$$

$$(ii) \quad x^3 + y^2 + x\sin y = \pi.$$

(b) If $y = x^2 \sin 2x$, find $\frac{d^{11}y}{dx^{11}}$ i.e. the eleventh derivative of y with respect to x.

(c) If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, where a is a constant, show

$$\frac{dy}{dx} = \tan t.$$

Find also $\frac{d^2y}{dx^2}$.

2. (a) Let $f(x, y) = x^{\ln y}$. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(b) Verify that

$$f(x, y) = xy + \frac{x}{y},$$

satisfies the equation

$$y\frac{\partial^2 f}{\partial y^2} + 2x\frac{\partial^2 f}{\partial x \partial y} = 2x.$$

(c) Let $y = \sinh^{-1} x$. Use $\frac{dy}{dx} = (\frac{dx}{dy})^{-1}$ to find $\frac{dy}{dx}$. Use integration by parts to find

$$\int \sinh^{-1} x.$$

3. Find the following integrals

(a)

$$\int \cot 2x dx,$$

(b)

$$\int \frac{1}{\sqrt{-x^2+4x}} dx,$$

(c)

$$\int \frac{3-3x}{2x^2+6x} dx,$$

(d)

$$\int_0^{\pi} \sin x \cos^2 x dx.$$

4. (a) Solve

$$\frac{dy}{dx} = y \sin x + 2xe^{-\cos x}, \quad y(0) = 1.$$

(b) Solve

$$(x^2-1)\frac{dy}{dx} = 2xy + 2x.$$

5. (a) Find the modulus and argument of $-1-i\sqrt{3}$. Hence find $(-1-i\sqrt{3})^{10}$ in $a+ib$ form. Find also the square roots of $-1-i\sqrt{3}$ in $a+ib$ form.

(b) Use complex numbers to find

$$\int e^{-x} \sin 3x dx.$$

6. (a) Define the dot and the cross product between two vectors **a** and **b**.

(b) Two unit vectors $\hat{\mathbf{c}}$ and $\hat{\mathbf{d}}$ are perpendicular. Find $(\hat{\mathbf{c}} \times \hat{\mathbf{d}}) \times \hat{\mathbf{c}}$.

(c) The three points (1, −2,1), (0, 2, 1) and (−1, 1, 2) form the vertices of a triangle. Use vector methods to find the angle between the two sides of the triangle which meet at (0, 2, 1). Find, also, using vector methods, the area of the triangle.

7. (a) Find the general solution of the differential equation

$$y'' + y' + 2y = 3x^2 + \cos x.$$

(b) Solve the following initial-value problem

$$y'' + 3y' + 2y = e^x, \quad y(0) = 1, \quad y'(0) = 0.$$

8. (a) Write down the first three non-zero terms in the MacLaurin series of the following functions

(i) $y(x) = \cos(x^2)$;

(ii) $y(x) = (1 + 2x)^{-1/2}$.

(b) Find the following limits:

$$(i) \quad \lim_{x \to 0} \frac{x \sin x - x^2}{x^4};$$

$$(ii) \lim_{n \to \infty} \frac{(2n + 3n^2)^3}{(n+1)^6}.$$

9. (a) Determine whether the following series are convergent or divergent, justifying your answer.

$$(i) \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{n^{1/2} + n^{1/4} + 1},$$

$$(ii) \quad \sum_{n=0}^{\infty} \frac{(n-1)^2}{n^4 + n^2 + 1}.$$

(b) Use Newton's method to find an approximate solution of the equation $4 \ln x = x$ to three decimal places starting from $x = 1$.

10. (a) Define the Poisson probability distribution with mean μ.

(b) Write down the binomial distribution for x successes in n independent trials each with probability p of success.

(c) On average, 0.15% of the nails manufactured at a factory are known to be defective. If a random sample of 400 nails is inspected, what is the probability of there being no more than 3 defective nails?