UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Eng. *M.Eng.*

Mathematics E001: Mathematics

COURSE CODE	:	**MATHE001**
UNIT VALUE	:	**0.50**
DATE	:	**10-MAY-05**
TIME	:	**10.00**
TIME ALLOWED	:	**3 Hours**

05-C0948-3-230

TURN OVER

All questions may be attempted but only marks obtained on the best **seven** *solutions will count. The use of an electronic calculator is permitted in this examination.*

1. (a) Let $f(x) = x^2 \cos x$. Find $\dfrac{d^{11} f}{dx^{11}}$.

   (b) A curve is given by the equation $x^2 - 2xy + 2y^2 = 10$. Find, implicitly, $\dfrac{dy}{dx}$ and hence find the equation of the normal at $y = 1$ which cuts the curve at a positive value of x.

   (c) Let $y = \cos t$ and $x = \cos 2t$. Use parametric differentiation to find $\dfrac{dy}{dx}$. Check your answer by using trigonometric identities to find a relation between y and x and then differentiating implicitly.

   Show
$$\frac{d^2 y}{dx^2} = -\frac{1}{16y^3}.$$

2. (a) Let $f(x, y) = e^x \cos(xy + 2)$. Find $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$ and $\dfrac{\partial^2 f}{\partial x \partial y}$.

   (b) Let $z = f(xy^2)$ where f is any differentiable function. Show that
$$2x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0.$$

   (c) Let $y = \sin^{-1} x$. Using $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$ find $\dfrac{dy}{dx}$. Find also
$$\int \sin^{-1} x \, dx.$$

3. (a) Find the following integrals:

   (i) $\displaystyle\int \frac{13x + 5}{3x^2 + 5x - 2} dx$,

   (ii) $\displaystyle\int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x} dx$,

   (iii) $\displaystyle\int \frac{dx}{\sqrt{4 + x^2}}$.

   (b) If
$$I_n = \int (\ln x)^n dx,$$
   show using integration by parts that
$$I_n = x(\ln x)^n - nI_{n-1}.$$
   Hence find
$$\int (\ln x)^3 dx.$$

4. Solve the following differential equations

(a) $(1+2y^2)\dfrac{dy}{dx} - y\cos x = 0, \qquad y(0) = 1.$

(b) $\cos x\dfrac{dy}{dx} + y\sin x = \frac{1}{2}\sin 2x, \qquad y(\pi) = 0$

5. (a) Express $-1+\sqrt{3}i$ in modulus-argument form. Evaluate $\left(-1+\sqrt{3}i\right)^8$, expressing your answer in $a+ib$ form.

Find also the square roots of $-1+\sqrt{3}i$ in $a+ib$ form.

(b) Use complex numbers to find

$$\int_0^\infty e^{-x}\cos 2x\ dx.$$

6. (a) Find the general solution of the differential equation

$$y'' - 2y' - 3y = \sin x.$$

(b) Solve the following initial-value problem

$$y'' + 4y = e^x, \qquad y(0) = 1, \qquad y'(0) = 0.$$

7. (a) Define carefully the dot and vector products of two vectors $\mathbf{a}$ and $\mathbf{b}$.

(b) Show, using the dot product, that if $\mathbf{c}-\mathbf{d}$ and $\mathbf{c}+\mathbf{d}$ are perpendicular then $|\mathbf{c}| = |\mathbf{d}|$.

(c) The vectors $\mathbf{a} = \mathbf{i}+2\mathbf{j}$ and $\mathbf{b} = \mathbf{i}-2\mathbf{j}+\mathbf{k}$ form two sides of a triangle. Use vector methods to find the area of the triangle and the angle between $\mathbf{a}$ and $\mathbf{b}$.

8. (a) Write down the first three non-zero terms in the Maclaurin series of the following functions

(i) $y(x) = \sin(3x)$;

(ii) $y(x) = e^{x+x^2}$.

(b) Find the following limits:

(i) $\lim_{x\to 0} \frac{1-e^{3x}}{2x}$;

(ii) $\lim_{n\to\infty} \frac{(n^2+3^n)^3}{3^{3n+1}+n^9}$.

9. (a) Determine whether the following series are convergent or divergent, justifying your answer.

(i) $\sum_{n=0}^{\infty} \frac{n+3}{n^3+3n+1}$,

(ii) $\sum_{n=0}^{\infty} \frac{n}{n+2^n}$.

(b) Use the trapezium rule to find an approximate value of the integral

$$\int_0^2 \sin x \, dx$$

by dividing the range of integration into five equal intervals. Compare your result with the exact answer.

10. (a) Write down the binomial distribution for x successes in n independent trials each with probability p of success.

(b) Write down the Poisson distribution with mean μ.

(c) What is the relation between the Poisson distribution and the binomial distribution?

(d) A factory produces identical items. On average, 0.2% of the items are known to be defective. If a random sample of 500 items is inspected, what is the probability of there being no more than 3 defective items?