UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

B.Sc.

Mathematics C396: Mathematics in Economics

COURSE CODE : **MATHC396**

UNIT VALUE : **0.50**

DATE : **09-MAY-06**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

06-C1027-3-50

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Suppose that $B = (b_{ij})$ is an $n \times n$ matrix such that $b_{ij} \leq 0$ for every $i \neq j$. Prove that the following statements are equivalent to each other.

   (i) There exists a vector $x \geq 0$ such that $Bx > 0$.

   (ii) For every non-negative vector c there exists a vector $x \geq 0$ such that $Bx = c$.

   (iii) B is invertible and the entries of B^{-1} are nonnegative.

   (iv) Each principal minor of B is positive.

2. Let A be a non-negative $n \times n$ matrix. Prove that there exists a non-negative real number $\lambda(A)$ such that

   (i) $\lambda(A)$ is an eigenvalue of A, and there is a non-negative eigenvector corresponding to $\lambda(A)$;

   (ii) If ω is any (real or complex) eigenvalue of A then $|\omega| \leq \lambda(A)$.

3. Define what it means to say that a subset of $\mathbb{R}^n$ is (i) open, and (ii) closed. Define the interior and the closure of a set $E \subset \mathbb{R}^n$. Prove that the point $x \in \mathbb{R}^n$ is in the interior of the set $E \subset \mathbb{R}^n$ if and only if $B(x,r) \subset E$ for some $r > 0$. Prove that the point $x \in \mathbb{R}^n$ is in the closure of the set $E \subset \mathbb{R}^n$ if and only if $B(x,r) \cap E \neq \emptyset$ for every $r > 0$.

   Decide, whether or not the following statements are true. If the statement is true for every n, prove it. If there it is false, give a counter-example.

   If $A \subset \mathbb{R}^n$ is convex, then its interior is also convex.

   If $A \subset \mathbb{R}^n$ is open, then its convex hull is also open.

   If $A \subset \mathbb{R}^n$ is convex, then its closure is also convex.

   If $A \subset \mathbb{R}^n$ $(n \geq 2)$ is closed, then its convex hull is also closed.

4. Let a and b be distinct points in $\mathbb{R}^n$. Define the perpendicular bisector of the segment $[a, b]$. Prove that the perpendicular bisector of the segment $[a, b]$ is a hyperplane.

   Define what it means to say that two subsets of $\mathbb{R}^n$ are strictly separated by a hyperplane.

   Prove that if A, $B \subset \mathbb{R}^n$ are disjoint non-empty closed convex sets and at least one of them is bounded, then they can be strictly separated by a hyperplane.

5. Define what it means to say that a subset of $\mathbb{R}^n$ is a closed half-space.

   Prove that in $\mathbb{R}^n$ every n-simplex is the intersection of $n + 1$ closed half-spaces.

   Define the barycentric coordinates of the point $x \in \mathbb{R}^n$ with respect to an n-simplex $S \subset \mathbb{R}^n$. Define what we mean by the carrier of the point x.

   State and prove Sperner's Lemma about triangulations of a simplex.

END OF PAPER