UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics C396: Mathematics in Economics

COURSE CODE	:	**MATHC396**
UNIT VALUE	:	**0.50**
DATE	:	**10-MAY-05**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

05-C0944-3-50

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (i) Prove that every bounded and monotonic sequence of real numbers is convergent.
   (ii) Define what it means that a point c is a point of accumulation of a set $H \subset \mathbb{R}$. Prove that every bounded and infinite set has a point of accumulation.
   (iii) Prove that every bounded sequence of real numbers has a convergent subsequence.

2. (i) Define the geometric and algebraic multiplicity of an eigenvalue of a matrix. Give an example of a matrix A and an eigenvalue λ of A such that the geometric multiplicity of λ is strictly less than the algebraic multiplicity of λ.
   (ii) Suppose that A and B are $n \times n$ matrices such that $0 \leq A \leq B$, $A \neq B$ and B is indecomposable. Prove that the Frobenius root of A is strictly smaller than the Frobenius root of B.
   (iii) Prove that if the matrix A is non-negative and indecomposable, then its Frobenius root has algebraic multiplicity one. (You may use, without proof, the fact that if $\phi_A(t)$ is the characteristic polynomial of A then

$$\phi'_A(t) = \phi_{A_1}(t) + \ldots + \phi_{A_n}(t),$$

   where $A_1, \ldots, A_n$ are the principal subdeterminants of order $n-1$ of A.)

3. (i) Prove that a set $F \subset \mathbb{R}^n$ is a hyperplane if and only if there are a nonzero linear function $\ell : \mathbb{R}^n \to \mathbb{R}$ and a real number b such that $F = \{x : \ell(x) = b\}$.
   (ii) Prove that in $\mathbb{R}^n$ every n-simplex is the intersection of $n+1$ closed halfspaces.
   (iii) Prove that in $\mathbb{R}^n$ every n-simplex has a non-empty interior.

4. (i) Prove that if a set $A \subset \mathbb{R}^n$ has at least $n+2$ points then there is a decomposition $A = A_1 \cup A_2$ such that $A_1 \cap A_2 = \emptyset$ and $\operatorname{co} A_1 \cap \operatorname{co} A_2 \neq \emptyset$.

   (ii) Prove that if $A_1, \ldots, A_k$ are convex sets in $\mathbb{R}^n$ such that $k \geq n+2$ and any $n+1$ of the sets $A_1, \ldots, A_k$ have a nonempty intersection, then $\bigcap_{i=1}^{k} A_i \neq \emptyset$.

   (iii) Prove that if $A_1, A_2, \ldots$ are closed, bounded and convex sets in $\mathbb{R}^n$ such that any $n+1$ of them have a nonempty intersection, then $\bigcap_{i=1}^{\infty} A_i \neq \emptyset$.

5. State and prove Brouwer's fixed point theorem. (You may use, without proof, Sperner's Lemma about triangulations of a simplex, and the fact that in $\mathbb{R}^n$ every closed, bounded and convex set with a non-empty interior is homeomorphic to an n-simplex.)