UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:–

B.Sc. *M.Sci.*

Mathematics C396: Mathematics in Economics

COURSE CODE	:	**MATHC396**
UNIT VALUE	:	**0.50**
DATE	:	**28–MAY–04**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours**

04–C0939–3–50

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Suppose that $B = (b_{ij})$ is an $n \times n$ matrix such that $b_{ij} \leq 0$ for every $i \neq j$. Prove that the following statements are equivalent to each other.

   (i) There exists a vector $x \geq 0$ such that $Bx > 0$.

   (ii) For every non-negative vector c there exists a vector $x \geq 0$ such that $Bx = c$.

   (iii) B is invertible and the entries of B^{-1} are nonnegative.

   (iv) Each principal minor of B is positive.

2. Define what it means that (i) a sequence of $n \times n$ matrices A_k converges to the matrix A; and (ii) the series $\sum_{k=1}^{\infty} A_k$ is convergent and its sum equals A. Prove that if $\sum_{k=1}^{\infty} A_k$ converges then $A_k \to 0$.

   Let A be a non-negative $n \times n$ matrix. Define what it means that A is productive. Prove that the following statements are equivalent to each other.

   (i) A is productive.

   (ii) $I - A$ is invertible and $(I - A)^{-1} \geq 0$.

   (iii) The infinite series $\sum_{k=0}^{\infty} A^k$ converges.

3. Define what it means to say that a subset of $\mathbb{R}^n$ is (i) open, (ii) closed. Prove that a set $H \subset \mathbb{R}^n$ is closed if and only if, whenever $x_k \in H$ and $x_k \to x$ then $x \in H$.

   Define what it means to say that a subset of $\mathbb{R}^n$ is convex. Define the convex hull of a subset of $\mathbb{R}^n$.

   Prove that the convex hull of a closed and bounded set is closed.

   Give an example of a closed set such that its convex hull is not closed.

4. Let $\mathcal{H}$ be a finite system of convex subsets of $\mathbb{R}^n$ containing at least $n+2$ sets. Prove that if any $n+1$ elements of $\mathcal{H}$ have a nonempty intersection then all elements of $\mathcal{H}$ have a nonempty intersection. (Any results which are quoted in the course of the proof must be clearly stated.)

5. State and prove Sperner's Lemma about triangulations of a simplex. Prove that if S is a simplex and $f : S \to S$ is continuous then there is a point $x \in S$ such that $f(x) = x$. (Any results which are quoted in the course of the proof must be clearly stated.)