UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

B.Sc.

Mathematics C396: Mathematics in Economics

COURSE CODE	:	**MATHC396**
UNIT VALUE	:	**0.50**
DATE	:	**28-MAY-03**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

03-C0933-3-30

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Let A be a non-negative $n \times n$ matrix. Prove that A has a non-negative eigenvalue. Prove also that if $\lambda(A)$ is the largest non-negative eigenvalue of A then $|\omega| \leq \lambda(A)$ for every real or complex eigenvalue ω of A.

2. Define what it means to say that a matrix A is decomposable. Prove that a non-negative $n \times n$ matrix is decomposable if and only if there are $x \in \mathbb{R}^n$ and $\mu \in \mathbb{R}$ such that $x \geq 0$, $x \neq 0$, $x \not> 0$, and $Ax \leq \mu x$. Outline a proof of the fact that if A is a non-negative and indecomposable $n \times n$ matrix then the linear space $\{x \in \mathbb{R}^n : Ax = \lambda(A)x\}$ has dimension 1. (Any results which are quoted in the course of the proof must be clearly stated.)

3. (i) State and prove Carathéodory's theorem for the positive hull of a subset of $\mathbb{R}^n$.
   (ii) State and prove Carathéodory's theorem for the convex hull of a subset of $\mathbb{R}^n$.
   (iii) Prove that $H \subset \mathbb{R}^n$ is a hyperplane if and only if there is a non-zero linear function $\ell(x) = a_1x_1 + \ldots + a_nx_n$ and there is a real number b such that $H = \{x : \ell(x) = b\}$.

4. What does it mean to say that two sets in $\mathbb{R}^n$ are strictly separated by a hyperplane? Prove that two disjoint convex and closed sets are strictly separated by a hyperplane provided that one of them is bounded. (Any results assumed in the proof must be clearly stated.) Give an example of two disjoint convex closed sets that are not strictly separated by any hyperplane.

5. Let K be a closed bounded convex set in $\mathbb{R}^n$, and let $F : K \to 2^K$ be a closed point-to-set mapping such that $F(x)$ is non-empty and closed for each $x \in K$. Prove that F has a fixed point; that is, a point $c \in K$ such that $c \in F(c)$. (Any results which are quoted in the course of the proof must be clearly stated.)