UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Sc. *M.Sci.*

Mathematics C396: Mathematics in Economics

COURSE CODE	:	**MATHC396**
UNIT VALUE	:	**0.50**
DATE	:	**29-APR-02**
TIME	:	**14.30**
TIME ALLOWED	:	**2 hours**

02-C0928-3-30

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Let A be a non-negative $n \times n$ matrix. Explain what it means to say that A is *productive*, giving also an economic interpretation. Describe an alternative condition for productivity of A, involving the leading principal minors of a related matrix.

   What does it mean to say that A is *profitable*, again giving an economic interpretation? Show that A is profitable if and only if it is productive.

   Determine, justifying the answers, which of the following matrices are productive.

$$\begin{bmatrix} 0.4 & 0.4 & 0.1 \\ 0.2 & 0.2 & 0.5 \\ 0.2 & 0.4 & 0.2 \end{bmatrix}, \quad \begin{bmatrix} 0.2 & 0.3 & 0.4 \\ 0.1 & 0.2 & 0.1 \\ 0.4 & 0.3 & 0.4 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0.7 & 0 \\ 0.7 & 0 & 0.6 \\ 0 & 0.6 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0.2 & 0.7 & 0 \\ 0.7 & 0 & 0.6 \\ 0 & 0.6 & 0.2 \end{bmatrix}.$$

2. State and prove Carathéodory's Theorem for the positive hull $\operatorname{pos} S$ of a subset S of $\mathbb{R}^n$.

   State Carathéodory's Theorem for the convex hull $\operatorname{conv} A$ of $A \subseteq \mathbb{R}^n$. Prove that, if A is closed and bounded, then $\operatorname{conv} A$ is also closed and bounded.

3. Define what it means to say that two convex sets K_0 and K_1 in $\mathbb{R}^n$ can be *properly separated.* State and prove a necessary and sufficient condition for K_0 and K_1 to be properly separable. (Any results assumed in the proof must be clearly stated.)

4. State and prove Sperner's Lemma about triangulations of a simplex (decompositions into subsimplices).

   Explain briefly how Sperner's Lemma is relevant to Brouwer's Fixed Point Theorem.

5. Let f be a convex function on $\mathbb{E}^n$, and let a be a point at which f is finite. Define the terms *directional derivative of f in direction y*, *subgradient of f at a*, and *subdifferential of f at a*. Prove that the directional derivative $f'(a; y)$ always exists. Prove also that $a^* \in \partial f(a)$ if and only if $\langle y, a^* \rangle \leqslant f'(a; y)$ for each $y \in \mathbb{E}^n$.