UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. B.Sc.(Econ)M.Sci.

Mathematics B611: Mathematics for Students of Economics, Statistics & Related Disciplines

COURSE CODE	:	**MATHB611**
UNIT VALUE	:	**0.50**
DATE	:	**21-MAY-03**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours**

03-C0907-3-130

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Explain what is meant by an elementary row operation, by an elementary matrix and by the reduced row echelon form of a matrix.

   (b) Find the reduced row echelon form of the matrix

$$A = \begin{pmatrix} 3 & 2 & 3 & -2 & 5 & 7 \\ 1 & 0 & 3 & 0 & 1 & -2 \\ 5 & 2 & 9 & -2 & 7 & 3 \\ 0 & 0 & 1 & 1 & 4 & -7 \\ 2 & 2 & 1 & -1 & 8 & 2 \end{pmatrix}.$$

   (c) What is the rank of A?

2. (a) Let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ be vectors in a vector space V. Explain what is meant by saying that

   (i) $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are linearly independent;

   (ii) $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ span a vector space V;

   (iii) $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ form a basis of V.

   (b) Let the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be linearly independent. Show that the following three vectors: $\mathbf{w}_1 = \mathbf{v}_2 - \mathbf{v}_1$, $\mathbf{w}_2 = \mathbf{v}_3 - \mathbf{v}_2$, $\mathbf{w}_3 = \mathbf{v}_1 + \mathbf{v}_3$ are linearly independent.

   (c) Find a basis for the vector space V consisting of all vectors $\begin{pmatrix} x \\ y \\ z \\ s \end{pmatrix} \in \mathbb{R}^4$ satisfying $x - y = z = -s$. What is the dimension of V?

3. (a) Give the definition of (i) a symmetric matrix and (ii) a skew-symmetric matrix.

   (b) Show that if A and B are skew-symmetric matrices then $C = AB^2 + B^2A + BAB$ is skew-symmetric.

   (c) Prove that every $n \times n$ matrix A can be expressed as a sum $A = B + C$ where B is symmetric and C is skew-symmetric. Show that such a decomposition is unique.

4. (a) Let A and B be $n \times n$ invertible matrices. Prove that AB is also invertible and find its inverse.

   (b) Let A be an $n \times n$ matrix. Show that A is invertible if and only if the rank of A equals n.

   (c) Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 3 & 0 & -1 \end{pmatrix}.$$

5. (a) Define the scalar product of two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ and the length of a vector $\mathbf{x} \in \mathbb{R}^n$.

   (b) State and prove the Cauchy-Schwarz inequality.

   (c) Assume $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, $|\mathbf{x}| = 1$, $|\mathbf{y}| = 2$, and $(\mathbf{x}, \mathbf{y}) = 1$. What is the length of $3\mathbf{x} - 2\mathbf{y}$? What is the cosine of the angle between $\mathbf{x} + 2\mathbf{y}$ and $\mathbf{x} - \mathbf{y}$?

6. (a) Explain how the Gram-Schmidt orthogonalization procedure works.

   (b) Compute the Gram-Schmidt orthogonalization of the vectors

$$\mathbf{a}_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{a}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{a}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

   (c) Assume that $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ is an orthogonal basis of $\mathbb{R}^n$. Let $\mathbf{x} \in \mathbb{R}^n$. Find the decomposition of $\mathbf{x}$ as a linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_n$.

7. (a) Define the determinant of an $n \times n$ matrix.

   (b) Show that an $n \times n$ matrix A is invertible if and only if its determinant is non-zero.

   (c) Let B be the following matrix:

$$B = \begin{pmatrix} 1 & 1 & -2 & 4 \\ 2 & 3 & 7 & -3 \\ 3 & 3 & -6 & -3 \\ 4 & 4 & 5 & -5 \end{pmatrix}.$$

   Compute its determinant. Are the rows of B linearly independent or not?

END OF PAPER