UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Sc. *B.Sc. (Econ)*

Mathematics B611: Mathematics for Students of Economics, Statistics & Related Disciplines

COURSE CODE	:	**MATHB611**
UNIT VALUE	:	**0.50**
DATE	:	**16-MAY-02**
TIME	:	**14.30**
TIME ALLOWED	:	**2 hours**

02-C0905-3-120

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Explain what is meant by an elementary row operation, by the row echelon form and the reduced row echelon form of a matrix.

   (b) The matrix
$$M = \begin{pmatrix} 1 & 0 & 4 & 1 & 0 & 7 \\ 0 & 2 & 0 & 6 & -4 & 2 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$
   has been obtained by elementary row operations from some matrix A. Find the reduced row echelon form of the matrix A.

   (c) What is the rank of A?

2. (a) Let $v_1, v_2, \ldots, v_n$ be vectors in a vector space V Explain what is meant by saying that

   (i) $v_1, v_2, \ldots, v_n$ are linearly independent;

   (ii) $v_1, v_2, \ldots, v_n$ span a vector space V;

   (iii) $v_1, v_2, \ldots, v_n$ form a basis of V.

   (b) Let the vectors $v_1, \ldots, v_n$ be linearly dependent. Show that at least one of these vectors can be written as a linear combination of the others.

   (c) Find a basis for the vector space V consisting of all vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$ satisfying $-3x = 5y = 7z$. What is the dimension of V?

3. (a) Give the definition of (i) an eigenvalue, and (ii) an eigenvector of an $n \times n$ matrix A.

   (b) Define the characteristic polynomial of A and show that λ is an eigenvalue of A if and only if λ is a root of the characteristic polynomial of A.

   (c) Find the eigenvalues and corresponding eigenvectors of the matrix
$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

4. (a) Assume A and B are n by n invertible matrices. Prove that AB is also invertible and find its inverse.

   (b) Let A be an $n \times n$ matrix. Show that A is invertible if and only if its reduced row echelon form is the identity matrix.

   (c) Find the inverse of the matrix

$$A = \begin{pmatrix} 2 & 1 & -3 \\ 1 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix}.$$

5. (a) Define the scalar product of two vectors $x, y \in \mathbb{R}^n$ and the length of a vector $x \in \mathbb{R}^n$.

   (b) State the Cauchy-Schwarz inequality. State and prove the triangle inequality.

   (c) Assume $x, y \in \mathbb{R}^n$, $|x| = 3$, $|y| = 1$, and $(x, y) = -1$. What is the length of $x - 2y$? What is the cosine of the angle between $x + y$ and $x - y$?

6. (a) Explain how the Gram-Schmidt orthogonalization procedure works.

   (b) Compute the Gram-Schmidt orthogonalization of the vectors

$$a_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \quad a_3 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}.$$

   (c) Assume that $v_1, v_2, \ldots, v_n$ is an orthogonal basis of $\mathbb{R}^n$. Let A be the $n \times n$ matrix whose kth column is v_k. Show that $A^T A$ is a diagonal matrix.

7. (a) Define the determinant of an $n \times n$ matrix.

   (b) Show that an $n \times n$ matrix A is invertible if and only if its determinant is non-zero.

   (c) Let B be the following matrix:

$$B = \begin{pmatrix} 2 & 3 & 1 & 1 \\ 2 & 0 & -1 & -2 \\ 0 & 2 & 0 & -3 \\ 2 & 0 & 1 & 1 \end{pmatrix}.$$

   Compute its determinant. Are the rows of B linearly independent or not?

END OF PAPER