UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *B.Sc.(Econ)LL.B.*

Mathematics B51B: Mathematics for Students of Economics, Statistics & Related Disciplines

COURSE CODE	:	**MATHB51B**
UNIT VALUE	:	**0.50**
DATE	:	**14-MAY-04**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

04-C0913-3-200

TURN OVER

Answer ALL questions from Section A.

All questions from Section B may be attempted, but only marks obtained on the best **two** *solutions from Section B will count.*

The use of an electronic calculator is **not** *permitted in this examination.*

Section A

1. (a) Maximize $f(x, y) = 2x^{1/4}y^{3/4}$, subject to the constraint $10x + 7y = 6$.

   (b) Minimize $f(x_1, x_2, x_3, x_4, x_5) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$, subject to the constraints $x_1 + x_2 + 3x_3 = 5$ and $x_2 + x_4 + x_5 = 3$.

2. (a) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x^2y,$$

   subject to the initial condition $y = 2$ at $x = 0$.

   (b) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y}{x} = e^{x^2},$$

   subject to the initial condition $y = 0$ at $x = 1$.

3. (a) Find the general solution of the following system of linear equations:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 &= 1 \\ x_1 - x_2 + x_3 - x_4 + x_5 &= 3 \\ x_1 + x_2 - x_3 - x_4 - x_5 &= 3. \end{aligned}$$

   (b) (i) For which values of λ is the matrix A invertible?

$$A = \begin{bmatrix} \lambda & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

   (ii) Find the inverse of A when $\lambda = 2$.

Section B

4. (a) Solve the difference equation

$$x_{n+2} - 5x_{n+1} + 6x_n = 4^{n+1},$$

subject to the initial conditions $x_0 = x_1 = 0$.

(b) Solve the difference equation

$$x_{n+2} - 3x_{n+1} + 2x_n = 10,$$

subject to the initial conditions $x_0 = x_1 = 0$.

5. (a) Give the solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 7\frac{\mathrm{d}y}{\mathrm{d}x} + 12y = e^{3x},$$

corresponding to the initial conditions $y = 1$ at $x = 0$ and $\frac{dy}{dx} = 0$ at $x = 0$.

(b) Give the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + y = 3x\cos 2x.$$

6. (a) Evaluate the integral

$$\int\int_R x^3 y \,\mathrm{d}A,$$

where R is the region of the plane satisfying $x \geqslant 0$ and $x \leqslant y \leqslant \sqrt{x}$.

(b) By changing to polar coordinates or otherwise evaluate the integral

$$\int_0^{5/\sqrt{2}} \int_y^{\sqrt{25-y^2}} (x^2+y^2)e^{-(x^2+y^2)^2}\,\mathrm{d}x\mathrm{d}y.$$

7. Evaluate the following integrals expressing your answers in terms of the gamma function. (Any identity you use relating the beta and gamma functions should be clearly stated.)

(a) $\displaystyle\int_0^1 \sqrt{\ln\left(\frac{1}{x}\right)}\,\mathrm{d}x,$ (b) $\displaystyle\int_0^{\pi/2} \sin^n x\,\mathrm{d}x,$ (c) $\displaystyle\int_0^1 \frac{\mathrm{d}x}{\sqrt{1-x^4}}.$

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END OF PAPER