UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A. *B.Sc.* *B.Sc.(Econ)*

Mathematics B51B: Mathematics for Students of Economics, Statistics & Related Disciplines

COURSE CODE	:	**MATHB51B**
UNIT VALUE	:	**0.50**
DATE	:	**06-MAY-03**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

TURN OVER

All questions may be attempted but only marks obtained on the best <u>five</u> *questions will count.*
The use of an electronic calculator is <u>**not**</u> *permitted in this examination.*

1. Find the maximum and minimum distances from $(0,0)$ to the ellipse with equation

$$x^2 - xy + y^2 + 2x + 2y - 5 = 0 \ .$$

2. Solve the following differential equations.

(a) $\dfrac{dy}{dx} = \dfrac{3x - 2y}{2x - 3y}$.

(b) $\dfrac{dy}{dx} - y\tan(x) = 4x^3\sec(x)$, with $y = 3$ at $x = 0$.

(c) $x^2\dfrac{dy}{dx} + xy - y^3 = 0$.

3. Solve the following differential equations.

(a) $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 4y = 32\,e^{2x} + 8\cos 2x$.

(b) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 10y = 10x\,e^{-x}$, with $y = \dfrac{4}{9}$ and $\dfrac{dy}{dx} = \dfrac{11}{3}$ at $x = 0$.

4. Solve the following difference equations.

(a) $x_n = 2x_{n-1} - \dfrac{1}{4}n^2 + 2^n - \dfrac{1}{2}$.

(b) $x_n - x_{n-1} = 2(x_{n-1} - x_{n-2}) + 2(1 - n)$, with $x_0 = 4$ and $x_1 = 10$.

5. (a) Calculate the integral

$$\int_0^1 \int_{x^5}^1 y \, dy dx \, .$$

Reverse the order of integration and again calculate the integral.

(b) By reversing the order of integration, calculate the integral

$$\int_0^1 \int_y^1 e^{x^2} \, dx dy \, .$$

(c) Prove that

$$\int_0^\infty e^{-x^2} \, dx = \tfrac{1}{2}\sqrt{\pi} \, .$$

6. Evaluate the following integrals, expressing your answers in terms of the *beta function* and *gamma function.*

(a) $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^4}} \, .$

(b) $\displaystyle\int_0^{\pi/2} \sqrt{\sin\vartheta} \, d\vartheta \, .$

(c) $\displaystyle\int_0^{\pi/2} \frac{d\vartheta}{\sqrt{1-\frac{1}{2}\sin^2\vartheta}} \, .$

7. Find the values of a and b for which the system of simultaneous linear equations

$$\begin{aligned} 2x + y + z &= a \\ x - 2y - z &= b \\ x + 3y + az &= 1 \end{aligned}$$

has

(a) no solution,

(b) more than one solution,

(c) just one solution.

Find the complete set of solutions in cases (b) and (c).

MATHB51B END OF PAPER