UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.A. *B.Sc.* *B.Sc. (Econ)* *M.Sci.*

Mathematics B51B: Mathematics for Students of Economics, Statistics & Related Disciplines

COURSE CODE	:	**MATHB51B**
UNIT VALUE	:	**0.50**
DATE	:	**13-MAY-02**
TIME	:	**14.30**
TIME ALLOWED	:	**2 hours**

02-C0904-3-160

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Find the extremal values of the function $x^2 + 2xy$ subject to the condition $2x^2 + y^2 = 1$.

2. Solve the following differential equations. Give the solution of (a) satisfying the given initial condition, and find the general solution of (b) and (c).

   (a) $xy' + 4y = x^4,\ y(1) = 1.$ (b) $y' + 2xy = e^{2x^2} \cdot y^3.$ (c) $x^2y' = xy + y^2.$

3. (a) Solve the following differential equation satisfying the given initial condition: $y'' - y' - 6y = 4e^{2x},\ y(0) = 0,\ y'(0) = 1.$

   (b) Find the general solution of the differential equation $y'' + 2y' + 2y = 2x.$

4. (a) Solve the following difference equation satisfying the given initial condition: $u_{n+2} - u_{n+1} - 12u_n = 5 \cdot 2^n,\ u_0 = 0,\ u_1 = 1.$

   (b) Find the general solution of the difference equation $u_{n+2} + 4u_{n+1} + 4u_n = 9n.$

5. Evaluate the following double integrals.

   (a)
   $$\int_0^1 \int_y^1 \sqrt[3]{1 + x^2}\, dx\, dy.$$

   (b) With R the domain $x^2 + y^2 \leq 4$ and $x \geq 0,\ y \geq 0$:
   $$\iint_R x \cdot \sqrt[3]{x^2 + y^2}\, dx\, dy.$$

6. Evaluate the following double integrals.

(a)
$$\int_0^\infty e^{-x^{2/5}}\,dx.$$

(b)
$$\int_0^1 \left(1-x^{2/3}\right)^{3/2}\,dx.$$

7. For what values of a and b does the system of linear equations

$$\begin{array}{rcrcrcl} ax & + & y & + & 3z & = & 1 \\ 2x & + & y & + & 2z & = & b \\ 4x & + & y & + & z & = & 3 \end{array}$$

have

(a) exactly one solution,

(b) more than one solution,

(c) no solution?

Find the complete set of solutions in cases (a) and (b).

8. Decide whether or not the following matrices have inverses. Find the inverse whenever it exists.

(a)
$$\begin{pmatrix} 2 & 1 & -1 \\ 0 & 3 & 2 \\ 1 & 4 & 1 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 1 & 3 & 2 \\ 2 & 4 & 1 \\ 1 & -1 & -4 \end{pmatrix}$$