UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A. *B.Sc.* *B.Sc.(Econ)M.Sci.*

Mathematics B51A: Mathematics for Students of Economics, Statistics & Related Disciplines

COURSE CODE	:	**MATHB51A**
UNIT VALUE	:	**0.50**
DATE	:	**08-MAY-06**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

06-C1001-3-200

TURN OVER

Answer ALL questions from Section A.
All questions from Section B may be attempted, but only marks obtained on the best **two** *solutions from Section B will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

Section A

1. (a) Use Gauss-Jordan or Gaussian elimination to find all solutions, if any, to the system:
   $2x_1 - 2x_2 + 3x_3 = 10$
   $x_1 + x_2 - x_3 = 2$
   $3x_1 - x_2 + 3x_3 = 5$

   (b) Use the elementary row operations to reduce the matrix A to row echelon form and reduced row echelon form:
   $$A = \begin{pmatrix} 1 & 1 & -1 \\ 3 & 2 & 1 \\ -2 & 3 & 1 \end{pmatrix}.$$

   (c) Find the determinate of the matrix A in part (b).

2. a) Show that $f_{xy} = f_{yx}$ for $f(x,y) = 2x^2 + 3x - 5xy + 2y - y^2$.

   b) Use differentials to approximate the change in
   $$w = 2x^2 - 4x^3y^2 + 2y^2$$
   if (x,y) changes from $(1,1)$ to $(1.01, 1.03)$.

   c) Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ for $w = 2u\cos 3v$, $u = 4x^2 + 5y^3$ and $v = 3x^3y^2$.

   d) If $z = f(x,y)$ satisfies the equation
   $$2x^2z^2 + 5xy^2 - 2z^2 + 6yz + 7 = 0,$$
   find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

3. State both the Comparison Test and the Limit Comparison Test for the convergence of infinite series. Use these comparison tests to determine whether the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{1}{n5^n}$

(b) $\sum_{n=1}^{\infty} \frac{7n^2-9}{2e^n(3n+1)^2}$

(c) $\sum_{n=1}^{\infty} \frac{3}{1+2\sqrt{n}}$

(d) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{2n^2+3}$

(e) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2+2}}$

Section B

4. (a) If f is a real valued function defined on an open interval containing a, define what it means for f to be differentiable at a.

(b) Differentiate the following functions:

(i) $f(x) = (1 - \sin x)(2x^5 + e^{\cos 2x})$;

(ii) $f(x) = \dfrac{\cos(x^2+1)}{\ln(x^2 - 2x + 4)}$.

(c) Assuming that the equation $x^5 + 4x^2y + y^2 = 32$ determines a function f such that $y = f(x)$, find y' and find the tangent line to the graph of $x^5 + 4x^2y + y^2 - 32 = 0$ at the point $P(2, 0)$.

5. (a) Let f be a real valued function on a closed interval $[a, b]$. State the Intermediate Value Theorem and the Mean Value Theorem for f.

(b) Verify the Intermediate Value Theorem for $f(x) = x^2 + 2x + 1$ on $[1, 2]$.

(c) For $f(x) = x^2 + 2x - 5$, verify the hypotheses of the Mean Value Theorem andfind all numbers c in $[0, 1]$ that satisfy its conclusion.

6. Evaluate the following integrals:

   (a) $\int x \cos 7x dx$

   (b) $\int \frac{x^3}{\sqrt{x^2+4}} dx$

   (c) $\int \frac{7x-15}{x(x-3)} dx$

   (d) $\int \frac{1}{x^2\sqrt{1-4x^2}} dx$ by means of hyperbolic substitution.

7. A rectangular box with no top and having a volume of 12 cubic metres is to be constructed. The cost per square metre of the material to be used is 4 pounds for the bottom, 3 pounds for two of the opposite sides, and 2 pounds for the remaining pair of opposite sides. Find the dimensions of the box that will minimize the cost.

END OF PAPER

Separate Handout for Mathematics B51A

Table of Integrals

Basic Forms

$$\int u\,dv = uv - \int v\,du$$

$$\int u^n\,du = \frac{1}{n+1}u^{n+1} + C, \quad n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u\,du = e^u + C$$

$$\int \cos u\,du = \sin u + C$$

$$\int \sin u\,du = -\cos u + C$$

$$\int \csc^2 u\,du = -\cot u + C$$

$$\int \sec^2 u\,du = \tan u + C$$

$$\int \csc u \cot u\,du = -\csc u + C$$

$$\int \sec u \tan u\,du = \sec u + C$$

$$\int \cot u\,du = \ln|\sin u| + C$$

$$\int \tan u\,du = \ln|\sec u| + C$$

$$\int \csc u\,du = \ln|\csc u - \cot u| + C$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$$

Hyperbolic Forms

$$\int \sinh u\,du = \cosh u + C$$

$$\int \cosh u\,du = \sinh u + C$$

$$\int \tanh u\,du = \ln\cosh u + C$$

$$\int \coth u\,du = \ln|\sinh u| + C$$

$$\int \operatorname{sech} u\,du = \tan^{-1}|\sinh u| + C$$

$$\int \operatorname{csch} u\,du = \ln\left|\tanh \tfrac{1}{2}u\right| + C$$

$$\int \operatorname{sech}^2 u\,du = \tanh u + C$$

$$\int \operatorname{csch}^2 u\,du = -\coth u + C$$

$$\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$$

$$\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$$