UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A. *B.Sc.* *B.Sc.(Econ)M.Sci.*

Mathematics B51A: Mathematics for Students of Economics, Statistics & Related Disciplines

COURSE CODE	:	**MATHB51A**
UNIT VALUE	:	**0.50**
DATE	:	**13-MAY-05**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

05-C0916-3-250

TURN OVER

Answer ALL questions from Section A.

All questions from Section B may be attempted, but only marks obtained on the best **two** *solutions from Section B will count.*

The use of an electronic calculator is **not** *permitted in this examination.*

Section A

1. (a) Find the sum of the series $\sum_{r=1}^{\infty} \frac{1}{3r(r+1)}$.

   (b) Determine whether or not the following series converge. Justify your answers.

   (i) $\sum_{r=1}^{\infty} \frac{r!}{10^r}$

   (ii) $\sum_{r=2}^{\infty} \frac{1}{r(\ln r)^p}$, for $0 \leqslant p \leqslant 1$ and $1 < p < \infty$

   (iii) $\sum_{r=1}^{\infty} \frac{r+1}{r^2+2}$

   (iv) $\sum_{r=1}^{\infty} \frac{1}{(r+\frac{1}{2})^2}$

   (c) Determine the values of $x > 0$ for which the series $\sum_{r=1}^{\infty} r^2 x^r$ converges.

2. If a closed rectangular box is to have a fixed volume V, find what relative dimensions will make the surface area a minimum.

3. (a) Use Gauss-Jordan or Gaussian elimination to find all solutions, if any, to the system:

   $x_1 - 2x_2 + 3x_3 = 11$

   $4x_1 + x_2 - x_3 = 4$

   $2x_1 - x_2 + 3x_3 = 10$

   (b) Use the elementary row operations to reduce the matrix A to row echelon form and reduced row echelon form:

   $$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & 3 \\ 5 & 6 & -2 \end{pmatrix}.$$

   (c) Find the determinate of the matrix A in part (b).

Section B

4. a) Show that $f_{xy} = f_{yx}$ for $f(x,y) = x^2 + 2x - 3xy + 7y - y^2$.

   b) Use differentials to approximate the change in

   $$f(x,y) = x^2 - 3x^3y^2 + y^2$$

   if (x,y) changes from $(1,1)$ to $(1.01, 1.03)$.

   c) Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ for $w = u\cos v$, $u = x^2 + y^3$ and $v = x^3y^2$.

   d) If $z = f(x,y)$ satisfies the equation

   $$x^2z^2 + 3xy^2 - z^2 + 8yz + 9 = 0,$$

   find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

5. (a) Evaluate the following integrals:

   (i) $\int_0^4 |x-3|dx$

   (ii) $\int 2xe^{-x}dx$

   (iii) $\int \frac{x^2}{2\sqrt{4-x^2}}dx$

   (iv) $\int \frac{4x^2+54x+134}{(x-1)(x+5)(x+3)}dx$

   (b) Evaluate the integral $\int \frac{3}{x^2(1-x^2)^{1/2}}dx$ by means of hyperbolic substitution.

6. (a) Show that $f(x) = 6x^2 - 24x + 22$ satisfies the hypotheses of Rolle's Theorem on $[0,4]$ and find all numbers c such that $f'(c) = 0$.

   (b) Determine whether the function f satisfies the hypotheses of the Mean Value Theorem on the indicated interval $[a,b]$ and if so, find all numbers c in (a,b) such that $f(b) - f(a) = f'(c)(b-a)$:

   (i) $f(x) = x^3 + 1$ on $[-2,2]$;

   (ii) $f(x) = \frac{3}{(x-1)^2}$ on $[0,2]$;

   (iii) $f(x) = |x-3|$ on $[0,4]$.

   (c) Find the absolute maximum and minimum of

   $$f(x) = 5 - 6x^2 - 2x^3 \text{ on } [-3,1].$$

7. (a) Differentiate the following functions

   (i) $f(x) = \left(\frac{5x+4}{2x-7}\right)^3$

   (ii) $g(x) = \left[(1+\frac{2}{x})^{-1}+2\right]^{-1}$

   (b) Assuming that the equation $x^4 + 3x^2y + 2y^2 = 8$ determines a function f such that $y = f(x)$, find y'.

   (c) Find the tangent line to the graph of $4x^3 - 2x^2y + 2y^3 - 2 = 0$ at the point $P(2, -3)$.