UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A. *B.Sc.* *B.Sc.(Econ)LL.B.*

Mathematics B51A: Mathematics for Students of Economics, Statistics & Related Disciplines

COURSE CODE : **MATHB51A**

UNIT VALUE : **0.50**

DATE : **06-MAY-04**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

04-C0912-3-200

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

A table of integrals is attached.

1. (a) State the 'Test for Extrema' for a function $f(x, y)$ of two variables that has continuous second partial derivatives on a rectangular region Q.
   (b) If $f(x, y) = 2x^2 - 8xy + 2y^3 + 8y$, find the extrema of f.
   (c) If an open rectangular box is to have a fixed volume V, find what relative dimensions will make the surface area a minimum.

2. (a) Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ for $w = 3ue^{2v}$, $u = x^2 + y^3$ and $v = x^4y^2$.
   (b) Show that $f_{xy} = f_{yx}$ for $f(x, y) = x^3 + x^2 - 3xy^2 - 3y^3$.
   (c) The radius and altitude of a right circular cylinder are measured as 2 cm and 5 cm, respectively, with a possible error in measurement of plus or minus 0.02 cm. Use differentials to approximate the maximum error in the calculated volume of the cylinder.

3. (a) Find the real and imaginary parts of the following complex numbers.
   (i) $\dfrac{5 - 9i}{3 + 3i}$
   (ii) $\dfrac{(1 + i)^{31}}{1 - i}$
   (b) Evaluate the following integrals.
   (i) $\displaystyle\int \frac{1}{x^2 - 6x + 5} dx$
   (ii) $\displaystyle\int \frac{x^2}{\sqrt{16 - x^2}} dx$
   (iii) $\displaystyle\int_1^6 |x - 5| dx$

4. (a) Evaluate the following limits (without using L'Hôpital's Rule).

   (i) $\lim_{x\to\infty} \frac{x+5}{x+3}$

   (ii) $\lim_{x\to 0} \frac{\sqrt{x+7}-\sqrt{7}}{x}$

   (iii) $\lim_{x\to\infty} \frac{1+\cos x}{2x^2}$

   (b) (i) Differentiate $f(x) = x^2$ from first principles.

   (ii) Differentiate the following functions.

   A. $f(x) = \ln \cos \frac{3}{x}$

   B. $f(x) = x^{2\cos 3x}$

5. (a) Let f be a real valued function on a closed interval $[a, b]$. State the Intermediate Value Theorem and the Mean Value Theorem for f.

   (b) Verify the Intermediate Value Theorem for $f(x) = 5x^2 + 2$ on $[0, 1]$.

   (c) For the function $g(x) = x + \frac{2}{x}$ on $[1, 2]$ determine whether it satisfies the hypotheses of the Mean Value Theorem on the interval $[1, 2]$; and, if so, find all numbers c in $(1, 2)$ that satisfy the Mean Value Theorem.

6. (a) State the limit comparison test for the convergence of infinite series.

   (b) Determine whether or not the following series converge or diverge, clearly stating any tests used.

   (i) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

   (ii) $\sum_{n=1}^{\infty} \frac{2}{\ln(n+1)}$

   (iii) $\sum_{n=1}^{\infty} \cos \frac{\pi n}{8}$

   (c) Find the following integrals.

   (i) $\int x \sin x dx$

   (ii) $\int_1^e \frac{\ln x}{4\sqrt{x}} dx$

7. Suppose that the demand for x units of a commodity is found to be related to the selling price of s pounds per unit by the equation $16s^2 + 5x - 30,000 = 0$. Find the demand function, the marginal demand function, the total revenue function, and the marginal revenue function. Find the number of units and the price per unit which will yield the maximum revenue.

END OF PAPER

Table of Integrals

Basic Forms

$$\int u\,dv = uv - \int v\,du$$

$$\int u^n\,du = \frac{1}{n+1}u^{n+1} + C, \quad n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u\,du = e^u + C$$

$$\int \cos u\,du = \sin u + C$$

$$\int \sin u\,du = -\cos u + C$$

$$\int \csc^2 u\,du = -\cot u + C$$

$$\int \sec^2 u\,du = \tan u + C$$

$$\int \csc u \cot u\,du = -\csc u + C$$

$$\int \sec u \tan u\,du = \sec u + C$$

$$\int \cot u\,du = \ln|\sin u| + C$$

$$\int \tan u\,du = \ln|\sec u| + C$$

$$\int \csc u\,du = \ln|\csc u - \cot u| + C$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$$

Hyperbolic Forms

$$\int \sinh u\,du = \cosh u + C$$

$$\int \cosh u\,du = \sinh u + C$$

$$\int \tanh u\,du = \ln\cosh u + C$$

$$\int \coth u\,du = \ln|\sinh u| + C$$

$$\int \operatorname{sech} u\,du = \tan^{-1}|\sinh u| + C$$

$$\int \operatorname{csch} u\,du = \ln\left|\tanh \tfrac{1}{2}u\right| + C$$

$$\int \operatorname{sech}^2 u\,du = \tanh u + C$$

$$\int \operatorname{csch}^2 u\,du = -\coth u + C$$

$$\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$$

$$\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$$