UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.A.   B.Sc.   B.Sc.(Econ)M.Sci.

Mathematics B51A: Mathematics for Students of Economics, Statistics & Related Disciplines

COURSE CODE   : **MATHB51A**

UNIT VALUE   : **0.50**

DATE   : **08-MAY-03**

TIME   : **14.30**

TIME ALLOWED   : **2 Hours**

TURN OVER

*All questions may be attempted but only marks obtained on the best **five** solutions will count.*
*The use of an electronic calculator is **not** permitted in this examination.*

1. Evaluate the following limits (without using L'Hôpital's Rule):

   a) $\lim_{x \to \infty} \frac{x+3}{x+2}$,

   b) $\lim_{x \to 0} \frac{\sqrt{x+5}-\sqrt{5}}{x}$,

   c) $\lim_{x \to 0} \frac{-1+\cos x}{2x^2}$,

   d) $\lim_{x \to 0} \frac{\cos x - \sec x}{\sin x}$.

2. a) Differentiate from first principles $\sqrt{9-x^2}$.

   b) Differentiate

      i) $\ln \cos \frac{3}{x}$,

      ii) e^{5x^7},

      iii) $x^{2\cos 3x}$.

   c) Find the n-th derivative of $\frac{1}{\sqrt{x+2}}$.

3. Find the following integrals:

   a) $\int x \sin x \, dx$,

   b) $\int \frac{x^{11}}{x^6-1} \, dx$,

   c) $\int \frac{1}{x^2-4x+3} \, dx$,

   d) $\int_0^{\pi/2} \frac{1}{(1+\cos x)^2} \, dx$,

   e) $\int_1^e \frac{\ln x}{2\sqrt{x}} \, dx$.

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4. Determine whether or not the following series converge or diverge, clearly stating any tests used.

   a) $\sum_{n=1}^{\infty} \frac{1}{2n(n+1)}$,

   b) $\sum_{n=1}^{\infty} \frac{n!}{3^n}$,

   c) $\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$,

   d) $\sum_{n=1}^{\infty} \cos \frac{\pi n}{4}$,

   e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}$.

5. a) Find the real and imaginary parts of the complex numbers:

      i) $\frac{2+3i}{1+i}$,

      ii) $(1+i)^{2i}$.

    b) Solve the equations:

      i) $z^3 + 1 = i\sqrt{2}$,

      ii) $(z+1)^3 + (z-1)^3 = 0$.

6. a) Show that $f_{xy} = f_{yx}$ for $f(x,y) = x^3 - 2x^2 - xy + y^3$.

    b) Use differentials to approximate the change in

$$f(x,y) = x^2 - 2x^2 y^3 + 5x - 2y^2 + 5$$

   if (x,y) changes from $(1,2)$ to $(0.98, 2.03)$.

    c) Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ for $w = u^2 \cos v$, $u = x^4 - 2y^4$ and $v = xy^3$.

    d) If $z = f(x,y)$ satisfies the equation

$$x^2 z^3 + xy^3 - z^3 + 4yz - 6 = 0,$$

   find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

7. a) Find the absolute maximum and minimum of

$$f(x) = x^3 - 5x^2 + 2 \ \ on \ \ [0, 2].$$

b) Find the critical numbers of the following functions and determine the local minima and local maxima.

    i) $f(x) = 1 - |x|^{1/3}$,

    ii) $f(x) = \frac{2x-3}{x^2-2}$.

c) For each of the following functions g determine whether it satisfies the hypotheses of the Mean Value Theorem on the indicated interval $[a, b]$. If so, find all numbers c in (a, b) such that $g(b) - g(a) = g'(c)(b - a)$.

    i) $g(x) = x + \frac{2}{x} \ \ on \ \ [1, 2]$,

    ii) $g(x) = \sqrt[3]{x^2} \ \ on \ \ [-1, 1]$.