UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics A3: Mathematics For Physical Science

COURSE CODE	:	**MATHA003**
UNIT VALUE	:	**0.50**
DATE	:	**18-MAY-06**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours**

06-C0992-3-30

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Prove that for any complex number z

   (i) $\text{Re}(iz) = -\text{Im}(z)$,

   (ii) $\text{Re}(z) = \dfrac{1}{2}(z + \bar{z})$, $\text{Im}(z) = \dfrac{1}{2i}(z - \bar{z})$.

   (b) Find the cartesian form of the equation

   $$|z - i| + |z + i| = 4,$$

   where $z = x + iy$. Sketch and identify the curve represented by the equation.

   (c) Find all of the roots of equation

   $$z^4 = -16,$$

   and draw their positions in the Argand diagram.

2. Using vectors prove that

   (a) the line joining the mid-points of two sides of a triangle is parallel to the third side and half its length;

   (b) the diagonals of a parallelogram bisect each other.

3. (a) Find the vector and parametric equations of the line L that passes through $P(2, 6, 1)$ and is parallel to $\underline{\mathbf{b}} = \underline{\mathbf{i}} + \underline{\mathbf{j}} + \underline{\mathbf{k}}$.

   (b) Find the vector and cartesian equations of the plane P that passes through the points $A(-2, 1, 1)$, $B(0, 2, 3)$, and $C(1, 0, -1)$.

   (c) Determine whether the line L and plane P are parallel or not. If they are not parallel, find the position vector of the point of intersection.

4. Calculate the following integrals, showing full workings in each case.

(a) $\displaystyle\int_0^2 x^3\sqrt{1+5x^4}\,dx,$ (b) $\displaystyle\int_0^{\frac{3}{2}} (9-4x^2)^{\frac{3}{2}}\,dx,$

(c) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{3}{1+\sin\theta}\,d\theta,$ (d) $\displaystyle\int_0^{\pi} \sin^5\left(\frac{\theta}{2}\right)\,d\theta.$

5. (a) Derive the MacLaurin series for $\sin x$ up to and including terms of order x^5.

(b) Use the series you found in (a) and the geometric series

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}, \quad |r| < 1,$$

to find the MacLaurin series up to and including terms of order x^5 for

$$f(x) = \frac{\sin x}{1-x^2}.$$

(c) Using Taylor's Theorem find out how many terms in the MacLaurin series for $\sin x$ are required to compute $\sin(0.1)$ with an error of less than 10^{-5}.

6. Solve each of the following systems of equations using Gaussian elimination, giving all solutions or showing there is no solution.

(a)
$$\begin{aligned} x_1 + x_2 - x_3 &= 7, \\ 4x_1 + 5x_2 + 5x_3 &= 4, \\ 6x_1 + 7x_2 + 3x_3 &= 18. \end{aligned}$$

(b)
$$\begin{aligned} x_1 - x_2 - 2x_3 &= -3, \\ x_1 \qquad + 2x_3 &= 3, \\ 2x_1 - 3x_2 - 7x_3 &= -11. \end{aligned}$$

(c)
$$\begin{aligned} 3x_1 + x_2 + 3x_3 &= 1, \\ x_1 + x_2 + 2x_3 &= 1, \\ 4x_1 + 2x_2 + 5x_3 &= 3. \end{aligned}$$

7. A predator-prey relationship can simplistically be modelled by the following differential equations

$$\frac{dx}{dt} = k_1 y, \quad \frac{dy}{dt} = -k_2 x,$$

where $x(t)$ represents the population of the predator species and $y(t)$ the population of the prey species, and k_1 and k_2 are positive constants.

(a) Show that $x(t)$ satisfies the equation of the harmonic oscillator

$$\frac{d^2x}{dt^2} + \omega^2 x = 0,$$

where $\omega = \sqrt{k_1 k_2}$.

(b) Derive the general solution of the equation of the harmonic oscillator.

(c) Suppose that $k_1 = k_2 = 1$ and initially the population of the predator species is 500 while that of the prey species is 2000. Find the populations of both species as a function of time if t is measured in weeks.

(d) When does the prey species become extinct under the conditions in (c)?

For the purposes of this question, you are given the following table of approximations to $\tan^{-1}$.

u	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
$\tan^{-1}(u)$	0.79	0.98	1.11	1.19	1.25	1.29	1.33	1.35	1.37	1.39