UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:–

M.Sci.

Mathematics A3: Mathematics For Physical Science

COURSE CODE : **MATHA003**

UNIT VALUE : **0.50**

DATE : **11–MAY–05**

TIME : **10.00**

TIME ALLOWED : **2 Hours**

05–C0907–3–30

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Calculate the following integrals, showing full workings in each case.

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{3-x^2}}\,dx, \qquad \int_0^{\frac{\pi}{2}} \frac{1}{2+\cos\theta}\,d\theta,$$

$$\int_0^{\frac{\pi}{4}} \sin^4\theta\,d\theta, \qquad \int_0^{1} (x^2-1)e^{-x}\,dx.$$

2. Find Maclaurin series up to and including x^5 for e^x, $\cos x$ and $\sin x$. Use these series to verify that

$$e^{i\theta} = \cos\theta + i\sin\theta$$

as far as terms in θ^5.

3. (a) State de Moivre's theorem, and then use it to derive expressions for $\cos 3\theta$ and $\sin 3\theta$ in terms of $\cos\theta$ and $\sin\theta$.

   (b) Find all of the roots of equation

$$\frac{z}{\bar{z}^2} = -1,$$

   and draw them on an Argand diagram.

4. (a) Find the vector and cartesian equations of the plane which passes through the points $(1,2,-1)$, $(2,3,1)$ and $(3,-1,2)$.

   (b) Find the point of intersection of the two lines

$$L_1: \quad \mathbf{r}_1 = (1,0,1) + t_1(1,2,3),$$
$$L_2: \quad \mathbf{r}_2 = (1,-2,1) + t_2(\tfrac{1}{3},1,1).$$

5. (a) Find the equation of the line in the (x, y)-plane which passes through the point $A = (2, 3)$ and which is parallel to the vector $5\underline{i} - 2\underline{j}$. Find the equation of the line in the (x, y)-plane which is perpendicular to the first line and also passes through the point A.

   (b) $OABC$ is a parallelogram with $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OC} = \underline{c}$. If D is the mid-point of OB, find the following vectors in terms of $\underline{a}$ and $\underline{c}$:

$$\overrightarrow{OB}, \quad \overrightarrow{DB}, \quad \overrightarrow{CD}, \quad \overrightarrow{DA}.$$

   Hence show that D is also the mid-point of AC.

6. An infectious disease is introduced into a population. Suppose $P(t)$ is the proportion of people exposed to the disease within t years of its introduction. If

$$\frac{dP}{dt} = \frac{2 - P}{4}$$

   and $P(0) = 0$, after how many years will 80% of the population be infected?(Assume $\ln \frac{5}{3} \approx 0.5$.)

7. Using Gaussian elimination determine the values of a and b for which the equations

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 2, \\ 2x_1 - x_2 + 2x_3 &= 3, \\ x_1 + x_2 + ax_3 &= b. \end{aligned}$$

   have

   (a) a unique solution,

   (b) no solutions,

   (c) an infinite number of solutions.

   In the cases (a) and (c), determine the solutions by back substitution.

END OF PAPER