# UNIVERSITY COLLEGE LONDON 

University of London

## EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-
B.A. B.Sc. M.Sci.

Mathematics A3: Mathematics For Physical Science

COURSE CODE : MATHA003

UNIT VALUE : 0.50

DATE : 12-MAY-03

TIME : $\mathbf{1 0 . 0 0}$

TIME ALLOWED : 2 Hours

All questions may be attempted but only marks obtained on the best five solutions will count.
The use of an electronic calculator is permitted in this examination.

1. (a) Find the two roots, $z_{1}$ and $z_{2}$ of the quadratic equation

$$
z^{2}+4 z+5=0
$$

and show that

$$
\frac{1}{z_{1}}+\frac{1}{z_{2}}=-\frac{4}{5}
$$

(b) Find all of the roots of equation

$$
4 z^{3}=\bar{z}^{2},
$$

and draw their positions in the Argand diagram.
2. (a) Find the equation of the line in the $(x, y)$-plane which passes through the point $A=(2,3)$ and which is parallel to the vector $5 \underline{i}-2 \underline{j}$. Find the equation of the line in the $(x, y)$-plane which is perpendicular to the first line and also passes through the point $A$.
(b) $O A B C$ is a parallelogram with $\overrightarrow{O A}=\underline{a}$ and $\overrightarrow{O C}=\underline{c}$. If $D$ is the mid-point of $O B$, find the following vectors in terms of $\underline{a}$ and $\underline{c}$ :

$$
\overrightarrow{O B}, \quad \overrightarrow{D B}, \quad \overrightarrow{C D}, \quad \overrightarrow{D A}
$$

Hence show that $D$ is also the mid-point of $A C$.
3. (a) Find the Cartesian equation of the plane passing through $P(2,6,1)$ and having $\underline{n}=(1,4,2)$ as a normal.
(b) Find the vector equation of the plane passing through the points $P_{1}(-2,1,1)$, $P_{2}(0,2,3), P_{3}(1,0,-1)$.
(c) Determine whether the line and plane are parallel:

$$
\begin{align*}
& x=4+2 t, \quad y=-t, \quad z=-1-4 t  \tag{1}\\
& 3 x+2 y+z-7=0 \\
& x=t, \quad y=2 t, \quad z=3 t  \tag{2}\\
& x-y+2 z=5
\end{align*}
$$

4. Calculate the following integrals, showing full workings in each case.
(a) $\int_{-5}^{0} \frac{1}{x^{2}+10 x+50} d x$,
(b) $\int_{0}^{1} \frac{1}{\sqrt{3-2 x-x^{2}}} d x$,
(c) $\int_{0}^{\frac{\pi}{2}} \frac{1}{3 \sin \theta+4 \cos \theta} d \theta$,
(d) $\int_{0}^{\frac{\pi}{4}} \sin ^{4} \theta d \theta$.
5. (a) State the $n$th degree Taylor polynomial for $f(x)$ in terms of $x-a$. State also the corresponding Taylor's Theorem.
(b) Derive the third degree Taylor polynomial for $\cos x$ when $a=\frac{\pi}{4}$.
(c) Compute $\cos \left(\frac{\pi}{4}+0.1\right)$ using the Taylor polynomial you get in (b), and estimate the error.
6. Using Gaussian elimination determine the values of $a$ and $b$ for which the equations

$$
\begin{aligned}
x_{1}-2 x_{2}+3 x_{3} & =2, \\
2 x_{1}-x_{2}+2 x_{3} & =3, \\
x_{1}+x_{2}+a x_{3} & =b .
\end{aligned}
$$

have
(a) a unique solution,
(b) no solutions,
(c) an infinite number of solutions.

In the cases (a) and (c), determine the solutions by back substitution.
7. A murder is committed. The temperature of the body is $37^{\circ} \mathrm{C}$ at the time of death. If $H(t)$ is the temperature of the body after $t$ hours, then $H$ satisfies

$$
\frac{d H}{d t}=-\frac{1}{16}(H-20) .
$$

The body is found at $4 \mathrm{p} . \mathrm{m}$. and its temperature is $30^{\circ} \mathrm{C}$. Approximately at what time was the murder committed?

