

UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

B.Eng.

Mathematics E004: Mathematics For Engineers

COURSE CODE : MATHE004

UNIT VALUE : 0.50

DATE : 23-MAY-06

TIME : 14.30

TIME ALLOWED : 2 Hours

All questions may be answered but only marks obtained on the best **five** solutions will count. The use of an electronic calculator is permitted in this examination.

1. Differentiate the following functions with respect to x .

(a) $\frac{x^3 + x^2 + 1}{x + \cos x}$,

(b) $e^{\sin x}$,

(c) $\sqrt{\log x}$,

(d) $\cos((x - 1)/x^2)$.

2. (a) Find the points where the curve $y = x^2 - 9x + 14$ meets the x -axis.

(b) Find the shortest distance between the line $7y + 2x = 53$ and the origin.

3. Write the following expressions in the form $a + ib$ where a and b are real numbers.

(a) $(3 + 4i)(5 + 5i)$,

(b) $\frac{(1 + 3i)(2 - 5i)}{(1 + i)}$

(c) $e^{i\pi/4}$,

(d) $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^8$.

4. (a) State the Binomial Theorem and use it to find the coefficient of x^{16} in $(1 + x)^{23}$.

(b) Write $f(x) = \sqrt{1 + x^2}$ as a power series (using Maclaurin's formula) up to the x^2 term.

5. Find the following:

(i) $\int \frac{1}{1+x^2} dx,$

(ii) $\int \log x \, dx,$

(iii) $\int \frac{x}{x^2-1} dx.$

6. a) Sketch the graph of $\tan x$. Explain how the inverse $\tan^{-1} x$ is defined and determine the derivative of $\tan^{-1} x$.

b) Determine whether the improper integrals (i) $\int_0^1 \log x \, dx$ and (ii) $\int_1^\infty \frac{1}{x^3} dx$ converge and, when convergent, determine the value.

7. a) Find the area under the curve $y = x^2(1-x)^2$ between $x = 0$ and $x = 1$.

b) Consider the curve $y = x(1-x)$ between $x = 0$ and $x = 1$. Find the volume of the body of revolution formed by rotating this curve about the x -axis.

8. By forming partial fractions, or otherwise, evaluate

(i) $\int \frac{4x+3}{x^2+5x+6} dx,$

(ii) $\int \frac{2x+1}{(x+2)^2} dx.$