UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:–

B.Eng. *M.Eng.*

Mathematics E004: Mathematics For Engineers

COURSE CODE	:	**MATHE004**
UNIT VALUE	:	**0.50**
DATE	:	**25–MAY–05**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

TURN OVER

All *questions may be attempted but only marks obtained on the best* **five** *solutions will count. The use of an electronic calculator is permitted in this examination.*

1. Simplify

   (a) $\dfrac{(x-y+z)(x^2-2xy+y^2+z^2)(x-y+2z)(x-y)}{(x-y)\left((x-y)^2+3z(x-y)+2z^2\right)}$,

   (b) $\left(\dfrac{(\sqrt[4]{y})^2 z^6}{\sqrt[4]{x}}\right)\dfrac{\sqrt[3]{y}}{\left(x^{\frac{1}{2}}zy^2\right)^3}$,

   (c) $x(x+y)^2-(x-2y)^3+(y-x)^3$.

2. Differentiate the following functions of x

   (a) $(x+1)^{\frac{1}{2}}\exp(x^2)$,

   (b) $\sin(\cos^2 x)$,

   (c) $\ln\dfrac{3x^3}{(x+1)(x+2)}$,

   (d) $\dfrac{\tan x}{x^2+1}$.

3. Write down Maclaurin's formula for expanding a function $f(x)$ as a series in **ascending** powers of x, and use it to show that, as far as the term in x^4,

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}\ldots \ .$$

   Show that $\ln\left(\frac{y}{y-1}\right) = -\ln\left(1-\frac{1}{y}\right)$ and hence write down the expansion for $\ln\left(\frac{y}{y-1}\right)$ as a series of **descending** powers of y as far as the term in y^{-4}. Set $y=5$ in your result to estimate $\ln(1{\cdot}25)$ correct to four decimal places.

4. A circle centre C has radius $5a$. A chord AB of length $3a$ is drawn. The tangents to the circle at A and B meet at T. Sketch the circle, chord and tangents.

   Denote the angle ACT by α and show that $\sin\alpha = 0{\cdot}3$. Find $\tan\alpha$ and the length AT.

   Show that the area enclosed by TA, TB and the **major** arc AB is $78{\cdot}8a^2$ correct to one decimal place.

5. Find the following

   (i) $\int \frac{x^2}{1+x^3}dx,$

   (ii) $\int x\cos x\ dx,$

   (iii) $\int x^{-\frac{2}{3}}\ln(x^{\frac{1}{3}})dx.$

6. Find the stationary values of the following functions and determine their nature

   (i) $y = 3x^4 - 8x^3 + 6x^2,$

   (ii) $y = x^2e^x.$

7. (i) Find the area under the curve $y = (1-x^2)^{-\frac{1}{2}}$ between $x = 0$ and $x = 1$.

   (ii) Find the volume of the solid of revolution obtained by rotating the curve $y = x(1-x)$ between $x = 0$ and $x = 1$ about the x-axis.

8. a) Solve the differential equation

$$\frac{dy}{dx} = \frac{y-1}{1+x^2}$$

given that $y = 4$ when $x = 1$.

b) Solve the differential equation

$$\frac{d^2y}{dx^2} + 16y = 0$$

given that $y = 1$ and $\frac{dy}{dx} = 4$ when $x = 0$. Find the amplitude of y.