

**EXAMINATION FOR INTERNAL STUDENTS**

*For The Following Qualifications:-*

*B.Eng. M.Eng.*

**Mathematics E004: Mathematics For Engineers**

COURSE CODE : **MATHE004**

UNIT VALUE : **0.50**

DATE : **12-MAY-03**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

All questions may be attempted but only marks obtained on the best five solutions will count.

The use of an electronic calculator is permitted in this examination.

1. Simplify the following expressions:

(a)  $(x - y)(x^3 + yx^2 + y^2x + y^3 + x + y)$ ,

(b)  $\left(\frac{x^3 \sqrt[3]{z}}{y}\right)^2 \left(\frac{1}{\sqrt{z}} y^{10}\right)$ ,

(c)  $(a + b + c)^2 - (a - b + c)^2$ .

2. (a) Find the points where the curve  $y = x^2 - 9x + 14$  meets the  $x$ -axis.  
(b) Find the equation of a straight line through the points  $(1, 13)$  and  $(-1, 7)$  and find the shortest distance of this line to the origin.

3. (a) Define the trigonometric functions  $\sin \theta$ ,  $\cos \theta$ ,  $\tan \theta$  and  $\cot \theta$  for a *general* angle  $\theta$ . Assuming the formula  $\sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi$  show that  $\sin(\theta + \pi/2) = \cos \theta$  and  $\cos(\theta + \pi/2) = -\sin \theta$ . Deduce the formula for expanding  $\cos(\theta + \phi)$  in terms of the functions  $\cos \theta$ ,  $\cos \phi$ ,  $\sin \theta$  and  $\sin \phi$

(b) Find exact values for the following:

(i)  $\cos 20\pi$ ,

(ii)  $\cos \frac{9\pi}{4}$ ,

(iii)  $\sin \frac{5\pi}{3}$ ,

(iv)  $\cos \frac{7\pi}{6}$ .

4. Differentiate the following with respect to  $x$ :

(a)  $(x^4 + x^2 + 2x + 3)(4x + 1)$ ,

(b)  $\frac{x^4 + 2x + 2}{x^2 + 3x + 4}$ ,

(c)  $\log x^3 + \cos x^2$ ,

(d)  $\cos(x - 1/x^2)$ .

5. Find the following:

(i)  $\int x e^{-2x} dx$ ,

(ii)  $\int \cos x / \sin^3 x dx$ ,

(iii)  $\int 1/(x^2 - 3x + 2) dx$ .

6. (a) Find the area enclosed between the positive  $x$ -axis and the curve  $y = x(1 - x^2)^3$ .

(b) Find the equation of the plane passing through the three points  $(1 \ 2 \ 1)$ ,  $(0 \ 1 \ -2)$  and  $(3 \ 1 \ -1)$ .

7. (a) Find the stationary points of the function  $y = x^5 - x^4$ , determining whether each point is a maximum, minimum or neither.

(b) Solve the differential equation

$$\frac{dy}{dx} = \frac{(y^2 + 1)}{xy}$$

given that  $y = 1$  when  $x = 1$ .

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END OF PAPER