UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

M.Eng.

Mathematics E004: Mathematics For Engineers

COURSE CODE	:	**MATHE004**
UNIT VALUE	:	**0.50**
DATE	:	**16-MAY-02**
TIME	:	**14.30**
TIME ALLOWED	:	**2 hours**

02-C0933-3-30

TURN OVER

All questions may be attempted but only marks obtained on the best **five** *solutions will count.*
The use of an electronic calculator **is** *permitted in this examination.*

1. Simplify the following expressions:

   (a) $c^2 - 2ab + (a+b-c)\,(a+b+c)$,

   (b) $\dfrac{\left(\frac{1}{\sqrt[5]{x}}\, y^6\, z^4\right)^3 z^{12}}{y^{18}\, x^3}$,

   (c) $\left(1 - x^{\frac{2}{3}}\right)\left(1 + x^{\frac{2}{3}} + x^{\frac{4}{3}} + x^2 + x^{\frac{8}{3}} + x^{\frac{10}{3}}\right)$.

2. (a) Write down the equation of the straight line through the points (x_1, y_1) and (x_2, y_2), where $x_1 \neq x_2$. What is the equation when $x_1 = x_2$? Find the equation of a straight line through the points $(4, 3)$ and $(6, 7)$ and find the shortest distance of this line to the origin.

   (b) Find the two points where the curves $y = 7x^2 - 24x + 10$ and $y = 8x^2 - 33x + 24$ meet and find the distance between these two points .

3. (a) Define the trigonometric functions $\sin\theta$, $\cos\theta$, $\tan\theta$ and $\cot\theta$ for a *general* angle θ. Sketch the graphs of $\cos\theta$ and $\sin\theta$ in the interval $-3\pi \leqslant \theta \leqslant 3\pi$.

   (b) By comparing the area of a certain sector of a circle with the area of two triangles, show that $\frac{1}{2}\sin\theta < \frac{1}{2}\theta < \frac{1}{2}\tan\theta$ for small values of $\theta > 0$. Complete the proof that $\dfrac{\sin\theta}{\theta} \to 1$ as $\theta \to 0$.

4. Differentiate the following with respect to x:

   (a) $(x^3 + x^2 + 5x + 3)(x + 3)$,

   (b) $\dfrac{x^3 + x^2 + 1}{x + \cos x}$,

   (c) $\sin(x^3)$,

   (d) $\exp(\sin(x^3))$.

5. Find the following:

(i) $\int x^2 e^{x^3}\, dx$,

(ii) $\int x \sin(2x)\, dx$,

(iii) $\int \frac{1}{x^2-1}\, dx$.

6. (a) Find the volume of the solid of revolution obtained by rotating the curve $y = \sqrt{\frac{x}{1+x^2}}$ between $x = 0$ and $x = 1$ about the x-axis.

(b) A body at the origin is acted on by a force of magnitude 1 in the direction of the point (1,2,1), a force of magnitude 2 in the direction of the point (-1,2,1), and a force of magnitude 3 in the direction of the point (0,0,1). Find the unit vector in the direction in which the body starts to move.

7. (a) Sketch the graph of $\tan x$. Explain how $\tan^{-1}(x)$ $(= \arctan(x))$ is defined and determine the derivative of $\tan^{-1}(x)$ from your definition.

(b) Solve the differential equation

$$\frac{dy}{dx} = \frac{\tan x}{\tan y},$$

given that $y = \pi/4$ when $x = 0$.