UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *B.Sc.(Econ)M.Sci.*

Mathematics M14B: Mathematical Methods 2

COURSE CODE	:	**MATHM14B**
UNIT VALUE	:	**0.50**
DATE	:	**05-MAY-05**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

05-C0960-3-210

TURN OVER

All questions may be attempted but only marks obtained on the best ***four*** *solutions will count.*
The use of an electronic calculator is ***not*** *permitted in this examination.*

1. (a) State, without proof, the general formula for a Fourier series on $(-L, L)$, with $L > 0$, for a function $f(x)$, giving the expressions for the coefficients.
   (b) On $(-\pi, \pi)$, find the Fourier series of $f(x) = |x|$.
   (c) State Parseval's identity.
   (d) Apply Parseval's identity to the function of part 1b to obtain an infinite series for a power of π.

2. (a) Using subscript notation, what is the expression for

$$\varepsilon_{ijk}\varepsilon_{klm}$$

   in terms of δ_{il}, δ_{jm}, etc.?
   (b) Using subscript notation, prove

$$\operatorname{curl}(\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - \mathbf{B}(\operatorname{div} \mathbf{A}) - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\operatorname{div} \mathbf{B}).$$

   (c) Verify the result of part 2b for

$$\mathbf{A} = (1, 0, 0), \qquad \mathbf{B} = (x, y, z).$$

3. (a) Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F} = (2x - y, 2y - x, 0)$ and C is the perimeter of the ellipse $x^2/9 + y^2/4 = 1$, $z = 0$, described in the anticlockwise sense.
   (b) Find $\int \mathbf{G} \cdot d\mathbf{r}$ for $\mathbf{G} = (y, z, yx)$ from $(0, 0, 0)$ to $(1, 1, 1)$ along
      (i) $\mathbf{r} = (t^2, t^3, t)$ with $0 \leq t \leq 1$,
      (ii) $\mathbf{r} = (t, t^2, t)$ with $0 \leq t \leq 1$.

   Is $\mathbf{G}$ conservative? Give your reasons for your conclusion.

4. (a) Define the Jacobian

$$\frac{\partial(x, y, z)}{\partial(u, v, w)},$$

where $x(u, v, w)$, $y(u, v, w)$ and $z(u, v, w)$ are smooth functions.

(b) Illustrate the cylindrical polar coordinate system by means of a sketch, and give the expressions for the Cartesian coordinates (x, y, z) in terms of the cylindrical polar coordinates.

(c) Determine the Jacobian, from your definition, for the change of coordinates from Cartesian coordinates to cylindrical polar coordinates.

(d) Find, by a using a suitable change of coordinates or otherwise,

$$\int_V (1 + z^3) \exp(x^2 + y^2)\, dV,$$

where V is the region $x^2 + y^2 \leq 1$ and $-1 \leq z \leq 1$.

5. (a) State the divergence theorem carefully.

(b) Given

$$\phi = \frac{1}{|\mathbf{r}|}, \qquad \mathbf{E} = -\operatorname{grad} \phi,$$

show that if $|\mathbf{r}| \neq 0$ then $\operatorname{div} \mathbf{E} = 0$.

(c) Let V_a be the ball of radius $a > 0$, and find the value of

$$I \equiv \int_{V_a} \operatorname{div} \mathbf{E}\, dV,$$

with $\mathbf{E}$ as defined in 5b, by first transforming this integral into a surface integral. Explain why the result of part 5b implies that the integral I does not depend on a.

6. (a) State Green's theorem carefully.

(b) Verify Green's theorem for the functions

$$P = xy, \qquad Q = (1 + x + y)^2,$$

and the region defined by

$$x \geq 0, \quad y \geq 0, \quad x + y \leq 1.$$

Sketch the region of integration.