UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics M14B: Mathematical Methods 2

COURSE CODE	:	**MATHM14B**
UNIT VALUE	:	**0.50**
DATE	:	**15-MAY-03**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

03-C0947-3-200

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) State, without proof, the general formula for a Fourier series on $(-L, L)$, with $L > 0$, for a function $f(x)$, giving the expressions for the coefficients.

   (b) On $(-\pi, \pi)$, find the Fourier series of $f(x) = x$.

   (c) Hence or otherwise, find the Fourier series of $g(x) = x^2$, on the range $(-\pi, \pi)$.

   (d) Hence or otherwise, show that

$$\frac{\pi^4}{90} = \sum_{n=1}^{\infty} \frac{1}{n^4},$$

   stating (without proof) any well known identity used.

2. (a) In the following, $r = |\mathbf{r}|$, where $\mathbf{r}$ is the position vector. Expressing your answers in terms of r and $\mathbf{r}$ as far as possible, determine the values of

$$\operatorname{grad} r, \quad \operatorname{grad} r^{-1}, \quad \operatorname{div}\operatorname{grad} r^{-1}, \quad \operatorname{div} \mathbf{r}, \quad \operatorname{curl}\operatorname{curl}(\mathbf{r} \times \mathbf{a}),$$

   where $\mathbf{a}$ is a constant vector.

   (b) Find

$$\phi = \operatorname{div}\operatorname{grad}(\exp(x)\sin(\lambda(y+z))\cos(\mu(y-z))).$$

   What is the condition on λ and μ for $\phi \equiv 0$?

3. (a) Find the arc length of the curve C given by

$$\mathbf{r}(t) = (\cos(t), \sin(t), t), \quad 0 \leqslant t \leqslant 2\pi.$$

   Sketch the curve C.

   (b) Find

$$I = \int_C \mathbf{F} \cdot d\mathbf{r},$$

   where C is defined in 3a and $\mathbf{F} = (y, x, x^2 + y^2)$.

   (c) Determine whether or not the vector field $\mathbf{F}$ defined in 3b is conservative.

4. (a) Define the Jacobian

$$\frac{\partial(x,y)}{\partial(u,v)},$$

where $x(u,v)$ and $y(u,v)$ are smooth functions. Show directly from your definition that

$$\frac{\partial(x,y)}{\partial(u,v)} = \left(\frac{\partial(u,v)}{\partial(x,y)}\right)^{-1}.$$

(b) Determine the Jacobian, from your definition, for the change of coordinates from Cartesian coordinates to plane polar coordinates.

(c) By a suitable change of coordinates, or otherwise, find

$$I = \int_0^\infty \int_0^\infty \exp\left(-a(x^2+y^2)\right)\, dx\, dy,$$

where a is a positive constant, and show in a diagram the region of integration.

5. (a) State carefully the divergence theorem.

(b) Expand

$$\operatorname{div}\left((\mathbf{J})(\mathbf{r}\cdot\mathbf{c})\right),$$

where $\operatorname{div}\mathbf{J} = \mathbf{0}$ and $\mathbf{c}$ is a constant vector.

(c) Hence, or otherwise, turn the following volume integral into a surface integral:

$$\int_V \mathbf{J}\, dV,$$

where $\operatorname{div}\mathbf{J} = \mathbf{0}$ and V is some bounded region of three-dimensional space with surface S.

(d) Verify your result for 5c in the case where $\mathbf{J} = \mathbf{k}$ and V is the unit ball.

6. (a) State carefully Stokes' theorem.

(b) State carefully Green's theorem in the plane.

(c) Show that Green's theorem in the plane is a special case of Stokes' theorem, defining all your symbols carefully.

(d) Verify Green's theorem in the plane for

$$\oint_C (y^3 dx + xy dy),$$

where C is the unit circle. *Hint: Use symmetry arguments where possible.*

 END OF PAPER