UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Sc.   *M.Sci.*

Mathematics M14B: Mathematical Methods 2

COURSE CODE	:	**MATHM14B**
UNIT VALUE	:	**0.50**
DATE	:	**03-MAY-02**
TIME	:	**14.30**
TIME ALLOWED	:	**2 hours**

02-C0942-3-180

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) State, without proof, the general formula for a Fourier series on $(-L, L)$, with $L > 0$, for a function $f(x)$, giving the expressions for the coefficients.

   (b) On $(-\pi, \pi)$, find the Fourier series of $f(x) = \exp(x)$.

   (c) Hence or otherwise, find the Fourier series of $g(x) = \sinh(x)$, and $h(x) = \cosh(x)$, each on the range $(-\pi, \pi)$.

2. (a) Find the directional derivative of $V = (x+y)^2 \exp(z)$ at the point $\mathbf{r}_0 = (1, 2, 3)$ in the direction $\mathbf{A} = (1, 1, 1)$.

   (b) Sketch the surface $z = 1 - (x^2 + y^2)^{\frac{1}{2}}$, $0 \leqslant z$. Find the tangent plane to the surface at the point on the surface with $x = y = \frac{1}{2}$.

   (c) Define the operators grad and curl. From your definitions, show that for any smooth scalar fields ϕ and ψ

$$\text{curl}\,(\phi\,\text{grad}\,\psi) = \text{grad}\,\phi \times \text{grad}\,\psi$$

   and verify your result for $\phi = \exp(x + y + z)$ and $\psi = z^2$.

   (d) Evaluate

$$\text{curl curl curl curl}\,(xyz + y^2 z,\ x^2 + yz,\ (x + y)^3).$$

   Explain your reasoning carefully, if you use any short cuts.

3. (a) Find the arc length of the curve C given by

$$\mathbf{r}(t) = (\cos(\cosh t), \sin(\cosh t), t), \quad 0 \leqslant t \leqslant a.$$

   Sketch the curve C for $a = \cosh^{-1}(2\pi)$.

   (b) Find

$$K = \oint_C \mathbf{F} \cdot d\mathbf{r},$$

   where C is the closed curve $\mathbf{r}(t) = (t(1-t),\ 2t(1-t),\ t^2(1-t))$, $0 \leqslant t \leqslant 1$, and $\mathbf{F} = \mathbf{j} \times \mathbf{r}$, where $\mathbf{j} = (0, 1, 0)$.

4. (a) Define the Jacobian

$$\frac{\partial(x,y)}{\partial(u,v)},$$

where $x(u,v)$ and $y(u,v)$ are smooth functions.

(b) Determine the Jacobian, from your definition, for the change of coordinates from Cartesian coordinates to plane polar coordinates.

(c) By a suitable change of coordinates, or otherwise, find

$$I = \int_0^\infty \int_0^\infty (x^2+y^2)^{\frac{3}{2}} \exp\left(-(x^2+y^2)^{\frac{5}{2}}\right)\, dx\, dy.$$

(d) By reversing the order of integration, find

$$J = \int_0^1 \int_y^1 x \exp(x^3)\, dx\, dy,$$

showing in a diagram the region of integration.

5. (a) State carefully the divergence theorem.

(b) Verify the divergence theorem for the vector field

$$\mathbf{A} = (x+yz)\mathbf{i} + (y^3 z + x)\mathbf{j} + (z + xyz)\mathbf{k}$$

and the region V enclosed by $x^2+y^2=1$ with $0 \leqslant z \leqslant 1$. Sketch the surface of V.

Hint: Use symmetry arguments where possible.

6. (a) State carefully Stokes' theorem.

(b) Verify Stokes' theorem for the vector field

$$\mathbf{A} = (1+xy^2)\mathbf{i} + (x + 2xyz + 2)\mathbf{j} + \sin(z)\mathbf{k},$$

and the surface S defined by $x^2+y^2+z^2=25$ and $z \geqslant 3$. Sketch the surface S.

Hint: Use symmetry arguments where possible.

END OF PAPER