UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Sc. *M.Sci.*

Mathematics M14A: Mathematical Methods 1

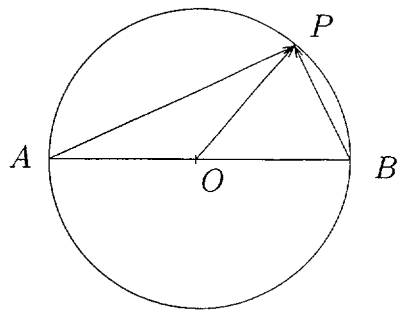
COURSE CODE	:	**MATHM14A**
UNIT VALUE	:	**0.50**
DATE	:	**08-MAY-02**
TIME	:	**14.30**
TIME ALLOWED	:	**2 hours**

02-C0941-3-180

TURN OVER

All questions may be answered, but only marks obtained on the best **four** *questions will count. The use of an electronic calculator is* **not** *permitted in this examination.*

1. (a) Define the scalar product $\mathbf{a} \cdot \mathbf{b}$ of the vectors $\mathbf{a}$ and $\mathbf{b}$. How can the scalar product be used to test if two vectors are normal to one another? Geometrically, when is $(\mathbf{a} \cdot \mathbf{b})^2 = a^2b^2$?

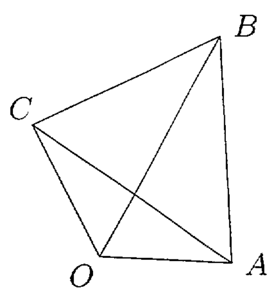


   (b) The diagram shows a circle radius a and centre O. Show that $\vec{AP} \cdot \vec{BP} = 0$ and deduce that the angle APB is a rightangle.

2. (a) Show that the area of the triangle AOB is $\frac{1}{2}|\mathbf{a} \wedge \mathbf{b}|$, where $\mathbf{a}$ and $\mathbf{b}$ are the position vectors, relative to O, of A and B respectively.

   (b) Three points A; B and C with position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ relative to O define a triangle ABC. Show that the area of the triangle is $\frac{1}{2}|\mathbf{a} \wedge \mathbf{b} + \mathbf{b} \wedge \mathbf{c} + \mathbf{c} \wedge \mathbf{a}|$.

   (c) In the diagram $OABC$ is a quadrilateral such that the diagonal OB bisects the diagonal AC.



   Show $\mathbf{a} + \mathbf{c} = 2t\mathbf{b}$, for some scalar t where $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are the position vectors of points A, B and C relative to O. Hence show that the line OB cuts the quadrilateral into two parts with equal area.

3. (a) State De Moivre's theorem and use it to evaluate

$$\left(1+\sqrt{3}i\right)^{10} - \left(1-\sqrt{3}i\right)^{10}.$$

(b) Find the n solutions of $z^n = 1$ with n a positive integer and plot them in the Argand diagram. Show that their sum is zero and hence deduce that

$$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}.$$

4. (a) Define $\sinh x$, $\cosh x$ in terms of the exponential function. Use these definitions to show that $\frac{d}{dx}\cosh x = \sinh x$ and $\cosh^2 x - \sinh^2 x = 1$.

(b) If

$$I_n = \int \cosh^n x \, dx, \quad n = 0, 1, 2, \ldots,$$

show that

$$nI_n = \sinh x \cosh^{n-1} x + (n-1)I_{n-2}, \quad n \geq 2,$$

and hence or otherwise evaluate

$$\int_0^x \cosh^4 u \, du.$$

5. (a) Write down the Taylor series expansion of the functions $\sin x$ and $\exp x$ about $x = 0$, giving the general term in each expansion.
Use these to show that

$$\exp(\sin x) = 1 + x + \tfrac{1}{2}x^2 - \tfrac{1}{8}x^4 - \tfrac{1}{15}x^5 + \cdots.$$

Hence write down the expansions of

$$\frac{1}{\exp(\sin x)} \quad \text{and} \quad \sinh(\sin x)$$

correct to terms in x^5.

(b) Evaluate the integrals

$$\int \frac{x^2+1}{x(x-4)^2} \, dx, \quad \int \frac{\cot(\ln x)}{x} \, dx.$$

6. (a) Show that the substitution $z(x) = (y(x))^{1-n}$, replacing the dependent variable $y(x)$ by $z(x)$, reduces the nonlinear equation

$$\frac{dy}{dx} + a(x)y = f(x)y^n$$

to the linear equation

$$\frac{dz}{dx} + (1-n)a(x)z = (1-n)f(x).$$

Hence show that the solution to

$$x\frac{dy}{dx} + y = x^4y^3, \quad y(1) = 1$$

is

$$y = \frac{1}{\sqrt{2x^2 - x^4}}.$$

(b) Solve the differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 16, \quad y(1) = \frac{dy}{dx}(1) = 0.$$