UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics C381: Logic

COURSE CODE : **MATHC381**

UNIT VALUE : **0.50**

DATE : **25-MAY-06**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

06-C1020-3-40

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Give the definition of a propositional language L and the set of L-formulas.
   (b) Define what it means for two formulas A and B to be logically equivalent.
   (c) Show that $(A \Rightarrow (B \vee C)) \equiv ((A \Rightarrow B) \vee (A \Rightarrow C))$.
   (d) Show that $((A \,\&\, B) \Rightarrow C) \not\equiv ((A \Rightarrow C) \,\&\, (B \Rightarrow C))$.

2. (a) Define a Hintikka set. State and prove Hintikka's Lemma for the Propositional Calculus.
   (b) Assuming that every infinite, finitely branching tree has an infinite path, prove that if S is a countable set of propositional formulas and each finite subset of S is satisfiable, then S is satisfiable.

3. (a) State and prove the Soundness Theorem for the Predicate Calculus. You may assume that if τ and τ' are tableaux such that τ' is an immediate extension of τ and τ is satisfiable, then τ' is satisfiable.
   (b) Fix a countable infinite set $V = \{a_1, a_2, \ldots, a_n, \ldots\}$ of parameters. Give the definition of a signed tableau. Use the signed tableau method and the Soundness Theorem to show that the sentence $(\exists x \forall y Rxy) \Rightarrow (\forall y \exists x Rxy)$ is logically valid.

4. (a) Give the definition of primitive recursive function, partial recursive function and recursive function.
   (b) Give the definitions of Turing machine, Turing program and Turing computable partial function.
   (c) Let $K = \{x \in \omega : \phi_x(x) \text{ converges}\}$. Note that ϕ_x denotes the x^{th} partial recursive function. Show that K is recursively enumerable, but not recursive.
   (d) Let $Tot = \{x \in \omega : \phi_x \text{ is a total function}\}$. Show that K is 1-reducible to Tot.

5. (a) In the language of arithmetic with a countably infinite set of variables $x, y, z, \ldots$ consider the k-place partial function $\lambda x_1 \cdots x_k[\psi(x_1, \ldots, x_k)]$. Define what it means for this function to be arithmetically definable.

(b) For each formula F in the language of arithmetic let $\sharp(F)$, be the Gödel number of F. Let $TrueSnt = \{\sharp S : S$ is a true sentence in the language of arithmetic$\}$. Show that every arithmetically definable set $A \subset \omega$ is reducible to $TrueSnt$.

(c) Supposing that every partial recursive function is arithmetically definable, show that the set $K = \{x \in \omega : \phi_x(x)$ converges$\}$ is arithmetically definable.

END OF PAPER