UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Mathematics C381: Logic

COURSE CODE : **MATHC381**

UNIT VALUE : **0.50**

DATE : **06-MAY-04**

TIME : **14.30**

TIME ALLOWED : **2 Hours**

04-C0932-3-40

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. (a) Describe what is meant by operational type, algebraic theory and the derivation of an equation in an algebraic theory.
   (b) In an algebraic theory with operational type $\{s_1, c_1, a_2, m_2\}$ find a generating sequence for the term $amsxcymcxsy$ and show that $axyamxaxy$ is not a term.
   (c) State the axioms of the Propositional Calculus and deduce $(p \Rightarrow r)$ from $\{(p \Rightarrow q), (q \Rightarrow r)\}$.

2. (a) In the Predicate Calculus a language L is a set of predicates with each predicate P of L being n-ary for some $n \in \omega$. Define the variables, quantifiers, L-U-formulas for a set U, the degrees of formulas and L-U-sentences.
   (b) Let U be a nonempty set and S a set of unsigned L-U-sentences. Define what it means for S to be open and to be U-replete.
   (c) State Hintikka's Lemma and use it, along with the Tableau Method, to conclude that the sentence

$$(\forall x \forall y \forall z((Rxy \& Ryz) \Rightarrow Rxz)) \& (\forall x \forall y(Rxy \Rightarrow \neg Ryx) \& (\forall x \exists y Rxy)$$

   is satisfiable in an infinite domain.

3. (a) Give the definition of primitive recursive function, partial recursive function and recursive function.
   (b) Give the definitions of Turing machine, Turing program and Turing computable partial function.
   (c) Assuming the Normal Form Theorem show that
      (i) there is a partial recursive function of two variables $\phi_z^{(2)}(e, n)$ such that $\phi_z^{(2)}(e, n) = \phi_e(n)$;
      (ii) there exists a 1 : 1 recursive function s of two variables such that for all variables x and y

$$\phi_{s(x,y)} = \lambda z[\phi_x^{(2)}(y, z)].$$

4. Let $K = \{x \in \omega : \phi_x(x) \text{ converges}\}$. A set $A \subset \omega$ is called an index set if for all x and y we have that $x \in A$ and $\phi_x = \phi_y$ implies that $y \in A$. Prove that if $A \neq \emptyset, \omega$, then either $K \leq_1 A$ or $K \leq_1 \bar{A}$.

5. Let $\mathcal{L}$ be a language, $\mathcal{E}$ a denumerable set of expressions of $\mathcal{L}$, $\mathcal{S} \subset \mathcal{E}$ the set of sentences of $\mathcal{L}$, $\mathcal{P} \subset \mathcal{S}$ the set of provable sentences of $\mathcal{L}$, $\mathcal{R} \subset \mathcal{S}$ the set of refutable sentences of $\mathcal{L}$, $\mathcal{H}$ a set of expressions called the predicates of $\mathcal{L}$, Φ a function that assigns to every expression E and every natural number n an expression $E(n)$ so that for every predicate H and every n, the expression $H(n)$ is a sentence, and a set $\mathcal{T}$ of sentences called the true sentences of $\mathcal{L}$. Let $\{E_n : E \in \mathcal{E}, n \in \omega\}$ be a Gödel numbering of $\mathcal{E}$, let $d(n)$ be the Gödel number of $E_n(n)$, and let $A^* = \{n \in \omega : d(n) \in A\}$.

   (a) Define what it means for a number set to be expressible in $\mathcal{L}$.

   (b) Define what it means for the system $\mathcal{L}$ to be correct.

   (c) Prove that if P denotes the set of Gödel numbers of all of the provable sentences, $\tilde{P}^*$ is expressible in $\mathcal{L}$ and $\mathcal{L}$ is correct, then there is a true sentence of $\mathcal{L}$ that is not provable in $\mathcal{L}$.

END OF PAPER