UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Sc. *M.Sci.*

Mathematics C381: Logic

COURSE CODE	:	**MATHC381**
UNIT VALUE	:	**0.50**
DATE	:	**21-MAY-02**
TIME	:	**10.00**
TIME ALLOWED	:	**2 hours**

02-C0922-3-70

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Describe the notions of (a) operational type, (b) algebraic theory, (c) derivation of an equation in an algebraic theory.

   In an algebraic theory with operational type $\{a_0, s_2\}$ and unique axiom $sax = x$

   (a) Give a derivation of $sxa = ssaxa$. For each line of the derivation explain which rule is being used and how.

   (b) Show that the equation $sxa = x$ is not provable.

2. In the propositional calculus, state the axioms and rules of inference and define the notion of a derivation.

   State and prove the Deduction Theorem of the propositional calculus. (In the proof, $\vdash (p \Rightarrow p)$ may be used without derivation.)

   Give a derivation to show that $\vdash (q \Rightarrow (p \vee q))$.

3. The sentence
$$(\forall x)(\forall y)(\forall z)((x = y) \vee (x = z) \vee (y = z)))$$
   is the single axiom of a first order theory with equality. Is the formula
$$(\forall x)(\forall y)(x = y)$$
   provable?

   State the axiom of induction of Peano Arithmetic (PA). In ZFC define the notion of ordinal successor. Define also the set ω of (von Neumann) natural numbers and state and show the principle of induction for ω.

   List all elements of the transitive closure of the set $\{\{\{\emptyset\}\}\}$ and explain, giving your reasons, which of them are (von Neumann) natural numbers and which are not.

4. Sketch the proof that recursive functions are computable. (An informal description of programs for the register machine suffices; details such as remembering initial values, clearing or copying registers may be skipped.)

   The function $h(m, n)$ is obtained from $f(m, n, k) = (m - kn)^2(m + 1 - (k + 1)n)^2$ by minimalization. What are (a) $h(62, 7)$ and (b) $h(63, 7)$?

5. Explain the notion of a $\mathcal{T}$-definable function where $\mathcal{T}$ is a first order theory containing PA. State a result connecting the notion of recursive functions and $\mathcal{T}$-definable functions.

   State Gödel's first incompleteness theorem and prove it using the Gödel-Rosser sentence. (The fixed point lemma may be used without proof.)