UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *B.Sc.(Econ)M.Sci.*

Mathematics M255: Linear Programming and Optimization

COURSE CODE	:	**MATHM255**
UNIT VALUE	:	**0.50**
DATE	:	**12-MAY-06**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours**

06-C1054-3-210

TURN OVER

All questions may be attempted but only marks obtained on the best **four** *solutions will count.*
The use of an electronic calculator is **not** *permitted in this examination.*

1. Let (P) be the linear program

$$\begin{array}{ll} \text{maximize} & x_1 + 2\,x_2 + 4\,x_3 \\ \text{subject to} & x_1, x_2, x_3 \geq 0 \\ & -x_1 - x_2 + x_3 \leq -4 \\ & x_1 + 2\,x_2 + 3\,x_3 \leq 10 \end{array}$$

Use the two-phase simplex algorithm to find an optimal solution to (P).

After each phase, identify the basic solution produced. Explain why the algorithm terminates. Give the value of the objective function at the optimal point.

2. a) Let (P) be the standard linear program

$$\begin{array}{ll} \text{minimize} & c^{\mathsf{T}} x \\ \text{subject to} & x \geq 0 \\ & A\,x \geq b \end{array}$$

Write down the dual program (P*). If x is feasible for (P) and y is feasible for (P*), show that

$$c^{\mathsf{T}} x \geq b^{\mathsf{T}} y.$$

b) A tableau for a canonical maximum program (P) is

0	0	2	1	0	-1	1
1	0	4	0	0	-3	5
0	0	-2	0	1	1	4
0	1	-1	0	0	-2	2
0	0	2	0	0	-3	7

Show that the objective function of (P) is unbounded. Give the basic feasible solution obtained when the algorithm terminates. In what direction v is the objective function unbounded?

3. a) Taking for granted that you know what a zero-sum game with payoff matrix A for two players Bob and Alice is, define the terms *mixed strategy* and *expected loss* for Alice.

b) Without turning it into the associated linear program, describe the minimization problem which Alice must solve for an optimal strategy t.

c) Find the value, and optimal mixed strategies for Bob and Alice, of the game whose payoff matrix is

$$\begin{bmatrix} 0 & 1 & -2 \\ 5 & 0 & 7 \end{bmatrix}$$

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Hint: Solve Alice's problem. Pivot on elements giving the largest increase in the objective function.

4. a) A transportation problem (T) is determined by its cost matrix (c_{ij}), supply vector (p_i) and demand vector (q_j).

Assuming that the total supply equals the total demand, write down the linear programs for (T) and (T*).

b) Consider the problem (T) whose data p_i, q_j, c_{ij} are given in the following array:

	50	20	10	35	15	50
30	5	10	15	8	9	7
40	14	13	10	9	20	21
10	15	11	13	25	8	12
100	9	19	12	8	6	13

Use the complementary slackness conditions to show that the *shipping matrix*

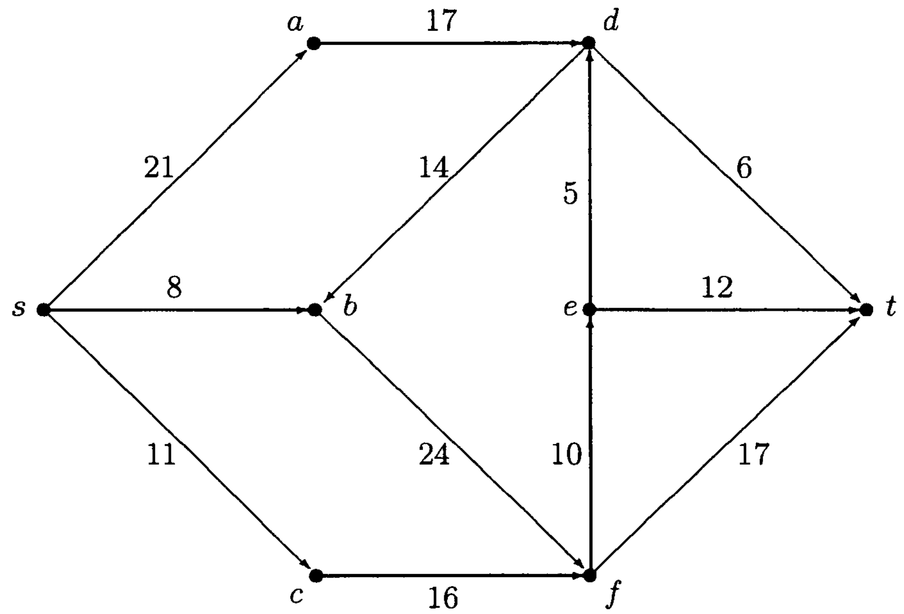
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 30 \\ 0 & 10 & 10 & 20 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 50 & 0 & 0 & 15 & 15 & 20 \end{bmatrix}$$

solves (T).

Hint: Choose one dual variable to be 0. Why are you allowed to do that?

5. a) Define the terms *flow*, *value of a flow*, *cut* and *capacity of a cut* in a network with capacities.

b) Using the Ford-Fulkerson algorithm, and starting with the zero flow, find a maximal flow and a minimal cut in the following network:



Explain the steps you take. Explain why the flow and the cut you found are optimal.

6. a) Define the *optimal assignment problem with aptitudes*, and outline the method employed in its solution.

b) Using the method, solve the assignment problem when the aptitude matrix is

$$\begin{bmatrix} 3 & 9 & 4 & 5 \\ 4 & 6 & 3 & 3 \\ 7 & 3 & 8 & 1 \\ 5 & 5 & 7 & 4 \end{bmatrix}$$

END OF PAPER